C2 JAN 2008 – SECT B.

(10) $V = 120\text{ cm}^3$; square base side x and height h

$\therefore x^2 h = 120 \Rightarrow h = \frac{120}{x^2}$

Surface area $\rightarrow$ 2 of x^2 + 4 of hx

$= 2x^2 + 4hx$

but $h = \frac{120}{x^2}$ So $A = 2x^2 + 4\left(\frac{120}{x^2}\right)x$

$A = 2x^2 + \frac{480}{x} = \left(2x^2 + 480x^{-1}\right)$

(ii) $\frac{dA}{dx} = 4x - 480x^{-2} = \underline{\underline{4x - \frac{480}{x^2}}}$

$\frac{d^2A}{dx} = 4 + 480 \times 2x^{-3} = \underline{\underline{4 + \frac{960}{x^3}}}$

(iii) $\frac{dA}{dx} = 0$ when $4x - \frac{480}{x^2} = 0$ $(\times x^2)$

$4x^3 - 480 = 0$

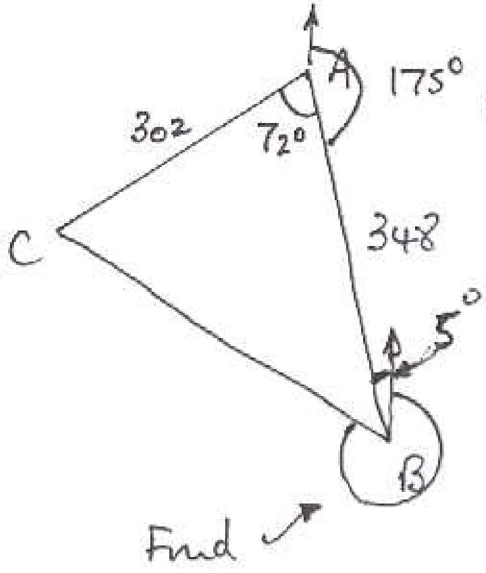
$x^3 = 120$ $x = \sqrt[3]{120}$

$\frac{d^2A}{dx^2} = 4 + \frac{960}{x^3}$, with $x^3 = 120$ $\frac{d^2A}{dx^2} = 4 + \frac{960}{120} = 12$

$\frac{d^2A}{dx^2} > 0 \Rightarrow$ MINIMUM when $x = \sqrt[3]{120}$ (4·93..)

$\therefore A = 2\left(\sqrt[3]{120}\right)^2 + \frac{480}{\sqrt[3]{120}} = 146\cdot0$ (4 S.F.)

(11)



(i) (A)

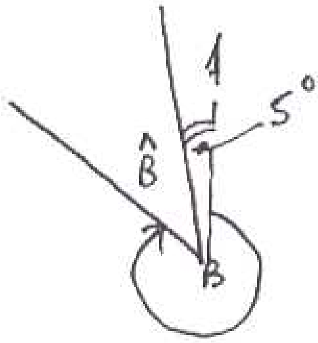
$$BC = \sqrt{302^2 + 348^2 - 2 \times 302 \times 348 \times \cos 72}$$

$$\therefore BC = 383.86\ldots \simeq 384\,m.$$

TOTAL DISTANCE $= 302 + 348 + 384$

$$= \underline{\underline{1034m}}$$

(i) (B)



Find $\hat{B}$

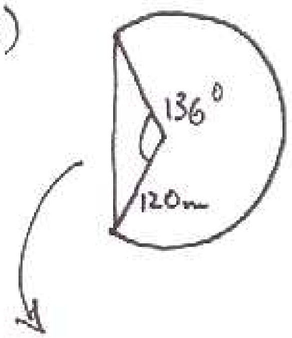
$$\frac{\sin B}{302} = \frac{\sin 72}{384}$$

$$B = \sin^{-1}\left(\frac{\sin 72 \times 302}{384}\right)$$

$$= 48.4 \simeq 48°$$

hence bearing at B $\quad 360° - (48+5)° = \underline{\underline{307°}}$.

(ii)



$r = 120m \quad$ angle of arc $= 360 - 136 = 224°$

$$\therefore \text{length of arc} = \frac{224}{360} \times 2 \times \pi \times 120$$

$$= 469.144\ldots$$

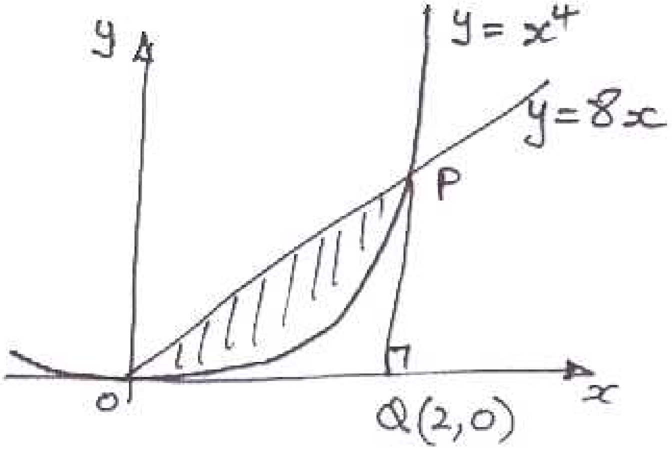
$$PQ = \sqrt{120^2 + 120^2 - 2 \times 120 \times 120 \times \cos 136°}$$

$$= 222.524\ldots$$

$\therefore$ TOTAL DISTANCE $\simeq 223 + 469 = \underline{\underline{692 \text{ miles}}}$.

(distances rounding to nearest mile).

(12)



Coords of P: $x^4 = 8x$

$x^4 - 8x = 0 \Rightarrow x(x^3 - 8) = 0$

$x = 0$ or $x^3 = 8$

(i) $\underline{x = 2}$ $\therefore Q(2,0)$

(A)

$x = 2 \quad y = 16 \qquad \underline{P(2, 16)}$

Area Δ OPQ $= \frac{1}{2} \times 2 \times 16 = \underline{\underline{16 \text{ sqn. units.}}}$

(i)

(B) Find Area of region (shaded)

$\Rightarrow$ Area of Δ OPQ $- \int_0^2 x^4 \, dx$

$= 16 - \left[\frac{x^5}{5}\right]_0^2$

$= 16 - \left(\left(\frac{32}{5}\right) - \frac{0}{5}\right) = 16 - 6\frac{2}{5}$

$= \underline{\underline{9\frac{3}{5}}}$ sqn units. (9.6 sqn units)

(12) contd.....

(ii) $f(x) = x^4$

(A) $f(x+h) = (x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

(B) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - \cancel{x^4}}{h}$

$= 4x^3 + \underbrace{6x^2h + 4xh^2 + h^3}$

(C) $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = 4x^3$ {as $h=0$ these terms add up to 0}

(D) gradient of [tangent to] curve.