Sect B (C2 - JUNE 2007)

9. $y = 2x^3 - 9x^2 + 12x - 2$

$\frac{dy}{dx} = 2 \times 3x^2 - 9 \times 2x + 12 = 6x^2 - 18x + 12$

$x = 3 \quad \frac{dy}{dx} = 54 - 54 + 12 = 12.$

$x = 3 \quad y = 54 - 81 + 36 - 2 = 7.$

$\therefore \ m = 12$ at $(3, 7)$

eqn $\quad y - 7 = 12(x - 3)$

$y = 12x - 29$

when $x = -1 \quad y = 12(-1) - 29 = -41$.

hence goes thro' point $(-1, -41)$

$\frac{dy}{dx} = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$

$= 6(x - 1)(x - 2)$

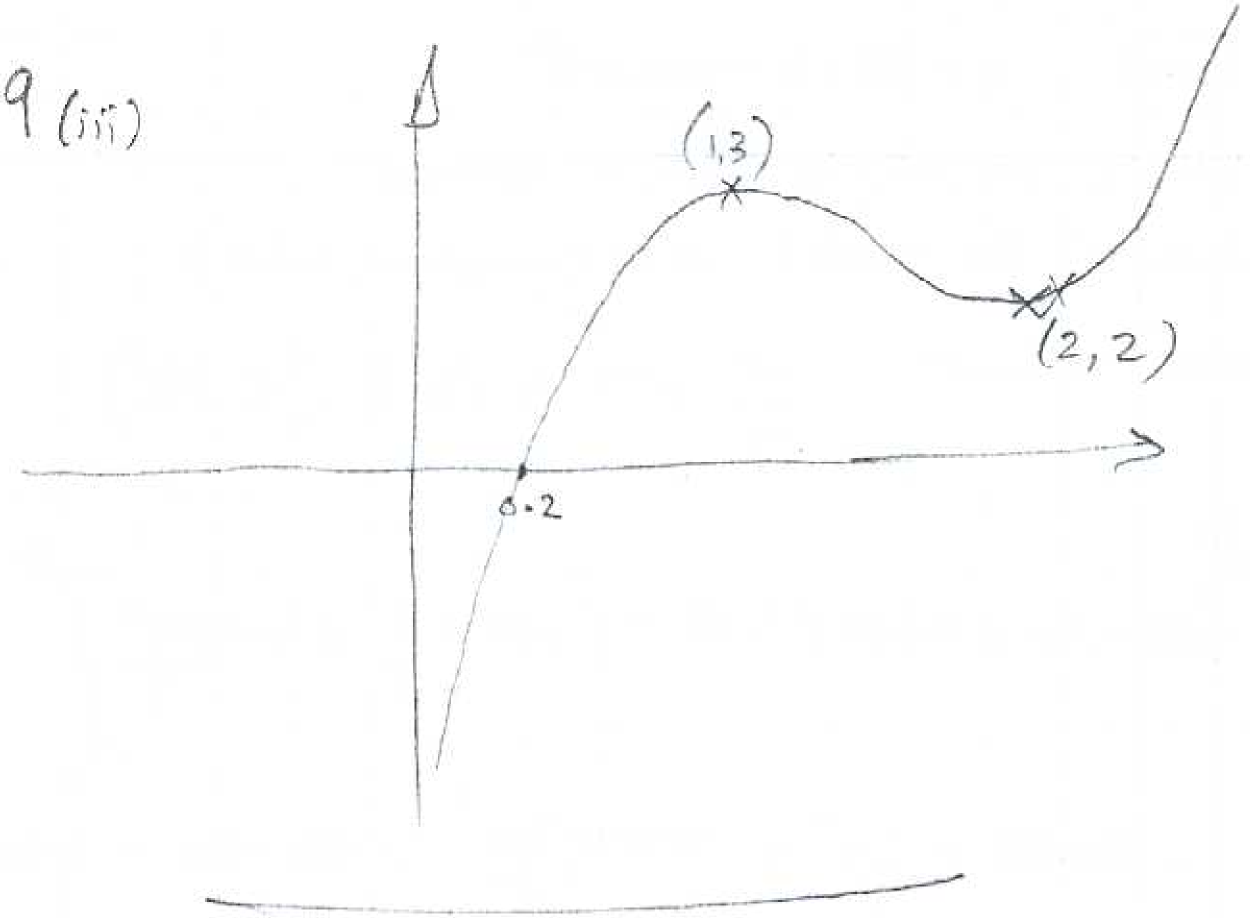
$\therefore \frac{dy}{dx} = 0$ when $x = 1$ and $x = 2$

$x = 1 \ y = 2 - 9 + 12 - 2 = 3$

$x = 2 \ y = 16 - 36 + 24 - 2 = 2$

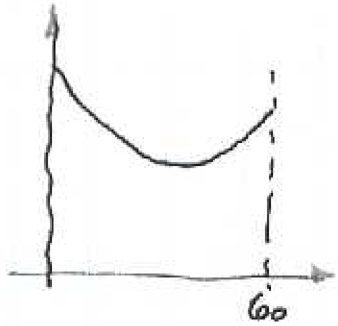
turning pts at $(1, 3)$ and $(2, 2)$

9 (iii)



10.

time (t) secs	0	10	20	30	40	50	60
Speed ($v\ ms^{-1}$)	28	19	14	11	12	16	22



(i) trapezium rule (6 strips)

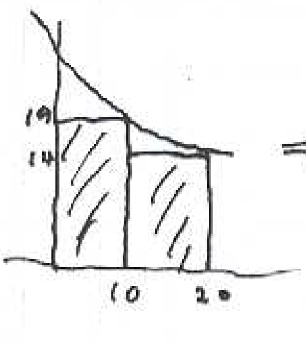
$h = 10$

$$\text{Area} = \frac{10}{2}\left(28+22+2\left(19+14+11+12+16\right)\right)$$

$$= 5 \times (50 + 144) = \underline{970\ m.}$$

(ii)

concave curve shows an area above the curve formed by the side of the trapezium
$\Rightarrow$ overestimate



$\Rightarrow$

$$\text{Area} = 19\times10 + 14\times10 + 11\times10 + 11\times10 + 12\times10 + 16\times10$$

$$= 93\times10 = \underline{930\,m}.$$ (6 rectangles).

10 (iii) Speed $v = 28 - t + 0.015t^2$

when $t = 10$ $v = 28 - 10 + 1.5 = 19.5 \text{ ms}^{-1}$

at $t = 10$ in the model $v = 19$ (measured value)

difference 0.5 ms^{-1} $\frac{0.5}{19} \times 100 = 2.63\% \ (< 3\%)$

(iv)
$$\int_0^{60} (28 - t + 0.015t^2)\, dt = \left[28t - \frac{t^2}{2} + 0.015\frac{t^3}{3} \right]_0^{60}$$

$$= 60 \times 28 - \frac{60^2}{2} + \frac{0.015(60)^3}{3} = 1680 - 1800 + 1080$$

$$= 960 \text{ metres}$$

11.(a) $3, 5, 7, 9, 11, \underline{13}$ counters in 6th pile.

number in 10^{th} pile is 21 counters

Sum of ten piles $\rightarrow$ $\frac{1}{2}n(a + l) = \frac{10}{2}(3 + 21)$

$= 5 \times 24 = \underline{120}$ counters

11 (b) $P_n = \frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1}$

(i) $P_4 = \frac{1}{6} \times \left(\frac{5}{6}\right)^3 = \underline{\underline{\frac{125}{1296}}}$

(ii) $P_1 = \frac{1}{6} \times \left(\frac{5}{6}\right)^0 = \frac{1}{6}$ $\left(a = \frac{1}{6}\right)$

$P_2 = \frac{1}{6} \times \left(\frac{5}{6}\right)^1 = \frac{5}{36}$ $\left(ar = \frac{5}{36}\right)$

$\therefore \quad r = \frac{ar}{a} = \frac{5/36}{1/6} = \frac{5}{6}$ $\quad a = \frac{1}{6}, \quad \underline{\underline{r = \frac{5}{6}}}$

$Sum_\infty = \frac{a}{1-r} = \frac{1/6}{1-5/6} = \frac{1/6}{1/6} = \underline{\underline{1}}$ (as reqd.)

(iii) $P_n < 0.001$, show n satisfies

$$n > \frac{\log_{10} 0.006}{\log_{10}\left(\frac{5}{6}\right)} + 1$$

$\frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1} < 0.001$ $\quad (\times 6)$

$\left(\frac{5}{6}\right)^{n-1} < 0.006$ $\quad$ (take logs)

$\log_{10}\left(\frac{5}{6}\right)^{n-1} < \log_{10} 0.006$

$n-1 \log_{10}\left(\frac{5}{6}\right) < \log_{10} 0.006$

Note. $\log \frac{5}{6} < 0$

$n-1 > \frac{\log_{10} 0.006}{\log_{10}(5/6)} \implies \underline{\underline{n > \frac{\log_{10} 0.006}{\log_{10} 5/6} + 1}}$