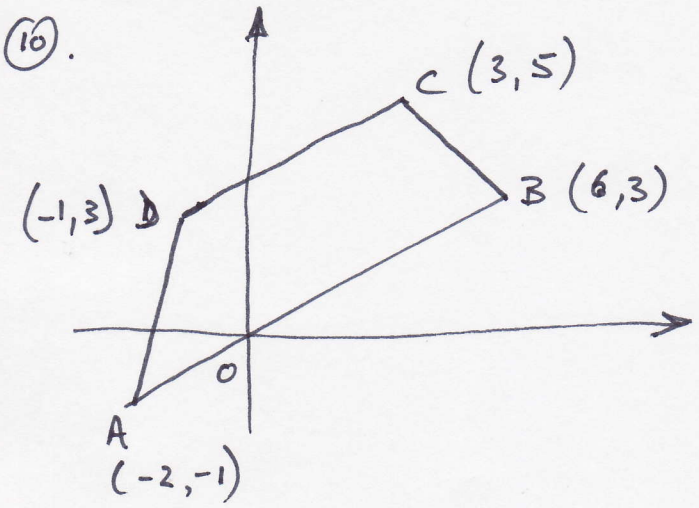
C1 Jan 2010 — SECT B

(10).



(i) AB // DC if gradients are equal

$$m_{AB} = \frac{(-1)-(3)}{(-2)-(6)} = \frac{-4}{-8} = \frac{1}{2}$$

$$m_{DC} = \frac{(3)-(5)}{(-1)-(3)} = \frac{-2}{-4} = \frac{1}{2}$$

$\therefore$ AB // DC

(ii) if AD $\neq$ BC then the trapezium is not isosceles.

length of AD is $\sqrt{((-1)^2 + (4)^2)} = \sqrt{(17)}$

length of CB is $\sqrt{((3^{\frac{1}{2}})^2 + (-2)^2)} = \sqrt{(13)}$

NOT ISOSCELES.

(iii) DB is the line $y = 3$

AC is the line with gradient $\frac{(5)-(-1)}{(3)-(-2)} = \frac{6}{5}$ thro' $(3,5)$

$\therefore \quad y - 5 = \frac{6}{5}(x-3)$

$5y - 25 = 6x - 18 \Rightarrow 6x - 5y = -7$

Subst $y = 3$ (the line DB) into $6x - 5y = -7$

$6x - 5(3) = -7$

$6x = 8 \quad x = 8/6 = 4/3$

hence $M\ (4/3, 3)$

10. (iv)

For neither diagonal to bisect each other....

point M cannot be the mid pt. of AC or BD.

mid pt. of AC is $\left(\frac{-2+3}{2}, \frac{-1+5}{2}\right) = \left(\frac{1}{2}, 2\right)$

mid pt of DB is $\left(\frac{-1+6}{2}, \frac{3+3}{2}\right) = (2\frac{1}{2}, 3)$.

with M $(4/3, 3)$ from pt(iii), neither diagonal bisects each other.

No.

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11. (i) circle $(x-3)^2 + (y+2)^2 = 25$

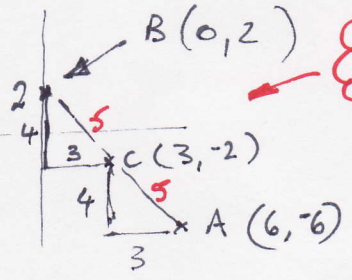
$\Rightarrow$ Centre $(3, -2)$; radius $\sqrt{25} = 5$.

(ii) A $(6, -6)$ lies on the circle?

$(6-3)^2 + (-6+2)^2 = 9 + 16 = 25$

So A does lie on the circle.

(ii) contd. if AB is a diameter then AC = CB (= 5); see diagram and points B, C, and A are in a straight line (a diameter).



{use of 3,4,5 Δ}

(iii) eqn of tangent at A? gradient of AC = $+\frac{4}{3} = -\frac{4}{3}$

$\therefore$ grad. of tangent is $\frac{3}{4}$

eqn of tangent with gradient $3/4$ thro' $(6, -6)$

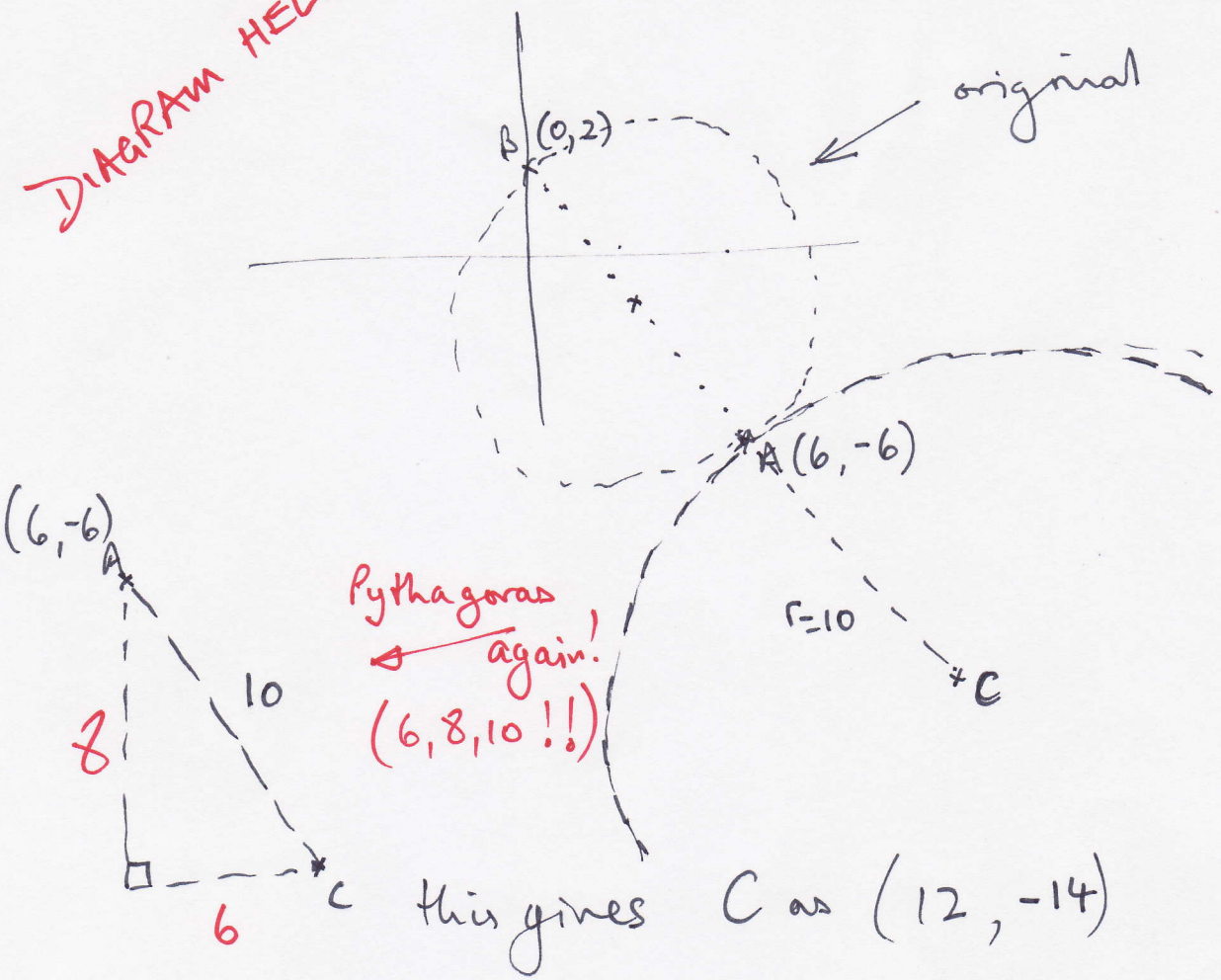
is $y - (-6) = \frac{3}{4}(x-6) \Rightarrow y + 6 = \frac{3x}{4} - \frac{18}{4}$ {× 4

$4y + 24 = 3x - 18$

$\Rightarrow \underline{3x - 4y = 42}$

11 (iv)

DIAGRAM HELPS!!



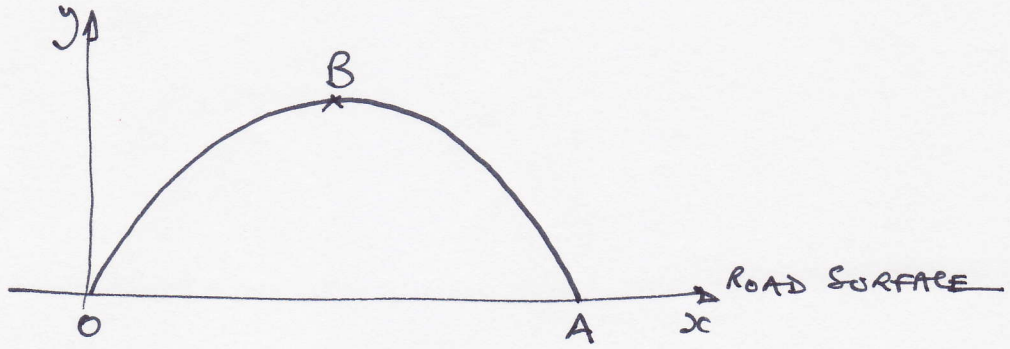
this gives C as (12, -14)

Eqn of second circle is

$$(x-12)^2 + (y+14)^2 = 100$$

(12) $y = \frac{1}{5}x(10-x)$ → models a bridge

(i)



Solving $\frac{1}{5}x(10-x) = 0$ $x = 0$ (the origin)

$x = 10$ (at pt A.)

(ii) by symmetry x coord. of B is 5

Subst $x = 5$ gives $y = \frac{1}{5} \times 5(10-5) = 5$

Hence B (max. pt.) is $(5, 5)$

Height of the arch is 5 metres.

NOTE: you could find B (a maximum) by differentiation

$y = \frac{x}{5}(10-x) \rightarrow y = \frac{10x}{5} - \frac{x^2}{5}$ $\therefore \frac{dy}{dx} = \frac{10}{5} - \frac{2x}{5}$

$\frac{dy}{dx} = 2 - \frac{2}{5}x$

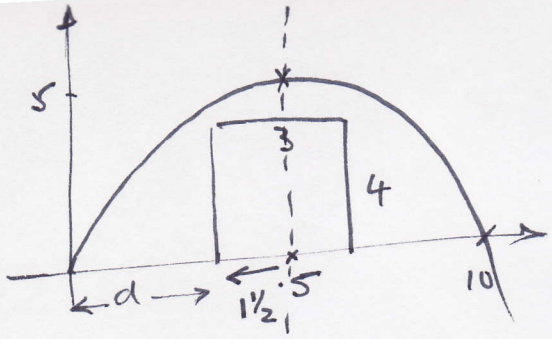
eqn $\frac{dy}{dx} = 0$ $2 - \frac{2}{5}x = 0 \rightarrow 2 = \frac{2}{5}x \rightarrow x = \frac{5 \times 2}{2}$

$x = 5$

Subst. into eqn for y gives $y = 5$

$\therefore$ height of bridge is 5m.

2 (iii)



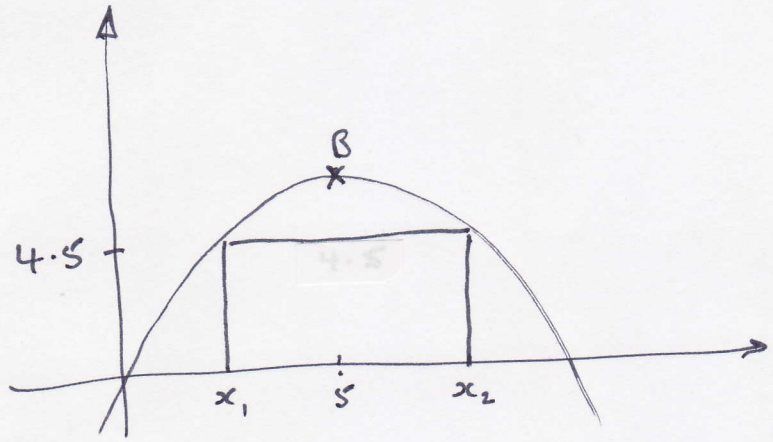
by symmetry, hence $d = 5 - 1\frac{1}{2}$
$= 3.5$ m.

ht(y) when $x = 3.5$ $\quad y = \frac{1}{5} \times 3.5\,(10 - 3.5)$

$= 0.7 \times 6.5 = \underline{4.55\text{m}}$

So lorry (4m high) will pass thro' the arch.

(iv)



Subst. $y = 4.5$ into eq^n $\quad 4.5 = \frac{x}{5}(10 - x)$

$$22.5 = 10x - x^2$$

$$x^2 - 10x + 22.5 = 0$$

Solving

$$x = \frac{-(-10) \pm \sqrt{100 - 4 \times 1 \times 22.5}}{2}$$

$$x = \frac{10 \pm \sqrt{10}}{2}$$

From diag.

$x_1 = \frac{10 - \sqrt{10}}{2}$ $\quad x_2 = \frac{10 + \sqrt{10}}{2}$ $\quad$ dist $x_2 - x_1 = \frac{(10 + \sqrt{10})}{2} - \frac{(10 - \sqrt{10})}{2} = \sqrt{10}$

WIDTH OF LORRY is $\sqrt{10}$ m.