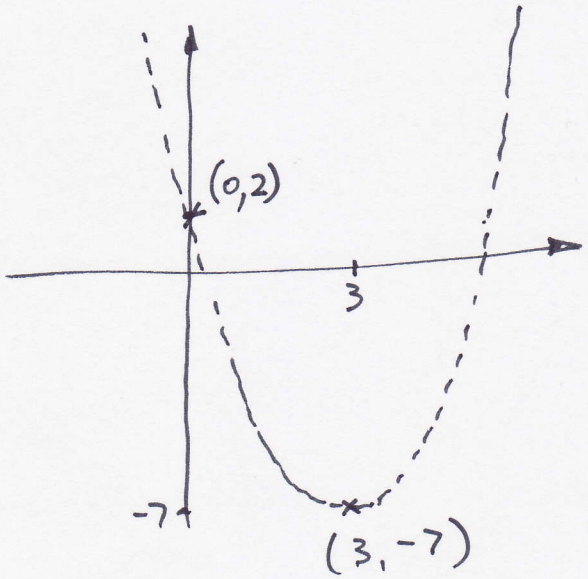
C1 JUNE 2008 – SECT B.

(10) (i) $x^2 - 6x + 2 = (x-3)^2 - 7$

(ii) $\therefore$ turning pt. of $y = x^2 - 6x + y$ is $(3, -7)$

(iii)



(iv) Solve $y = x^2 - 6x + 2$ and $y = 2x - 14$

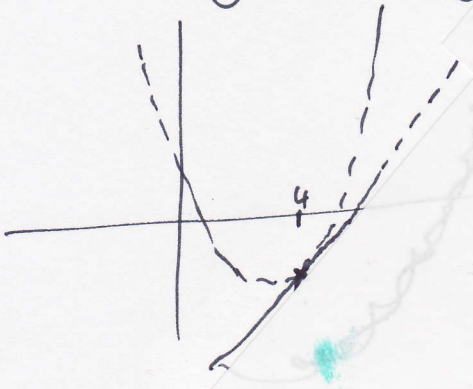
$\therefore \quad x^2 - 6x + 2 = 2x - 14$

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0 \quad \Rightarrow \quad x = 4$

(only one solution for x; when $x = 4$ the point $(4, -6)$ satisfies both eqns.

Hence $y = 2x - 14$ is a tangent to $y = x^2 - 6x + 2$ at $(4, -6)$



(11) $f(x) = 2x^3 + 7x^2 - 7x - 12$

(i) $f(-4) = 2(-4)^3 + 7(-4)^2 - 7(-4) - 12$

$= -128 + 112 + 28 - 12 = 0$

hence $x = -4$ is a root

Note { $(x+4)$ is a FACTOR }

(ii) $2x^3 + 7x^2 - 7x - 12 = (x+4)(2x^2 + px - 3)$

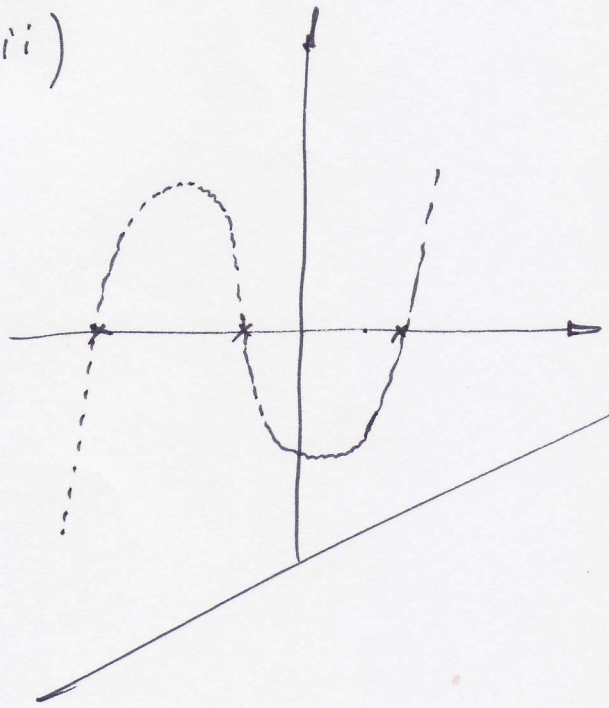
eqn coeffs of x: $-7 = -3 + 4p$

$-4 = 4p \rightarrow p = -1$

$\therefore f(x) = (x+4)(2x^2 - x - 3)$

$f(x) = (x+4)(2x-3)(x+1)$

(iii)



$\therefore$ NOTE: $x = -4, \quad x = \frac{3}{2} \quad x = -1$

(iv) show $f(x-4)$ ....

$f(x-4) = 2(x-4)^3 + 7(x-4)^2 - 7(x-4) - 12$

$= 2(x^3 - 12x^2 + 48x - 64) + 7(x^2 - 8x + 16)$
$-7x + 28 - 12$

$= 2x^3 - 24x^2 + 7x^2 + 96x - 56x - 7x$
$-128 + 112 + 28 - 12$

$\therefore f(x-4) = 2x^3 - 17x^2 + 33x$

12 (i) line passing thro' $A(-1, 1)$ & $B(3, 9)$

gradient $\frac{9-1}{3-(-1)} = \frac{8}{4} = 2$

gradient 2 thro' $(3, 9)$ $\quad y - 9 = 2(x - 3)$

$y = 2x + 3$

(ii) mid pt of AB $= \left(\frac{-1+3}{2}, \frac{1+9}{2} \right) = (1, 5)$

gradient of lr bisector $-1/2$ (as grad. of AB is 2)

Hence eqn of lr bisector of AB is

$y - 5 = -1/2 (x - 1)$

$2y - 10 = -1(x - 1)$

$2y + x = 11$

(iii) Circle has centre $(5, 3)$ and eqn $(x-5)^2 + (y-3)^2 = k$

Circle goes thro' $A(-1, 1)$ show that $k = 40$

$(-1-5)^2 + (1-3)^2 = (-6)^2 + (-2)^2 = 40 \quad \Rightarrow k = 40$

B is pt $(3, 9)$

$\therefore (3-5)^2 + (9-3)^2 = (-2)^2 + (6)^2 = 40$

Hence B is also on the circle.

12(iv) Circle is $(x-5)^2+(y-3)^2=40$

Circle ~~crosses~~ x axis $\rightarrow$ $y=0$

$$(x-5)^2+(0-3)^2=40 \Rightarrow (x-5)^2=31$$

$$\therefore x=5\pm\sqrt{31}$$