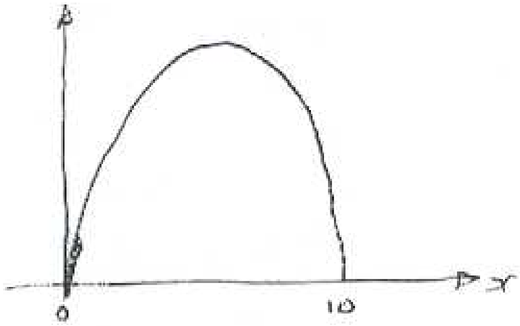
C2 Jan 2005 – SECT B.

9 (i) A tunnel 100m long has a cross section modelled by

$$y = \frac{1}{4}\left(10x - x^2\right)$$



(A) Find greatest height

by symmetry, greatest ht. when $x = 5$

$\therefore \text{height}_{max} = \frac{1}{4}\left(10 \times 5 - 5^2\right) = \underline{6{\cdot}25\text{m}}$

you would not have to use differentiation!

(B) $100\int_0^{10} y\,dx$ gives the volume because

$\int_0^{10} y\,dx$ is the Cross Sectional Area ; multiplying by the length 100 gives the volume.

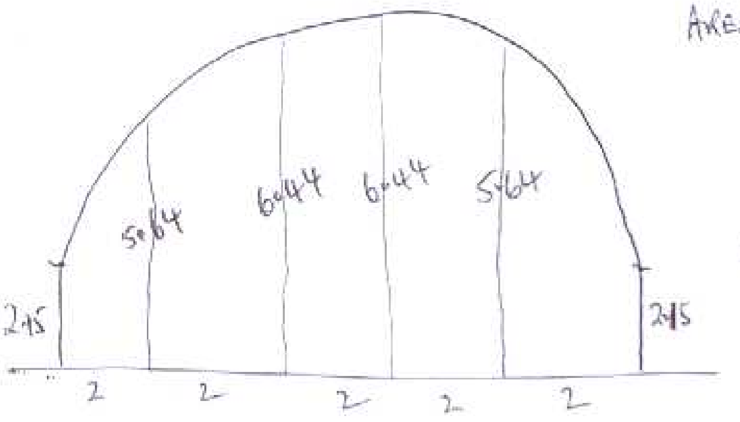
i.e. $100\int_0^{10} y\,dx$ = length × C.S.A.

$$100\int_0^{10} \frac{1}{4}\left(10x - x^2\right)dx = 25\int_0^{10}\left(10x - x^2\right)dx = 25\left[5x^2 - \frac{x^3}{3}\right]_0^{10}$$

$$= 25 \times \left(500 - \frac{1000}{3}\right) = 4166{\cdot}\dot{6}\ \text{m}^3$$

$$\simeq \underline{4167\ \text{m}^3}$$

(ii) Roof is re-shaped



$\text{AREA} \approx \frac{2}{2}\left[2{\cdot}15 + 2{\cdot}15 + 2\left(5{\cdot}64 + 6{\cdot}44 + 6{\cdot}44 + 5{\cdot}64\right)\right]$

$\approx 52{\cdot}62\ \text{m}^2$

Approx Vol $\Rightarrow$ $5262\ \text{m}^3$

$\therefore$ Earth removed on re-shaping

$5262 - 4167 \simeq \underline{1095\ \text{m}^3}$

(10) Curve has eqn $y = x^3 - 6x^2 + 12$

(i) use calculus to find turning pts ...

$$\frac{dy}{dx} = 3x^2 - 12x = 3x(x-4)$$

$\frac{dy}{dx} = 0$ when $x = 0, 4$

Subst. into $x^3 - 6x^2 + 12 \rightarrow 0^3 - 6(0)^2 + 12 = 12$

$\rightarrow 4^3 - 6(4)^2 + 12 = -20$

turning pts. at $(0, 12)$ and $(4, -20)$

$\frac{d^2y}{dx} = 6x - 12 = 6(x-2)$ $\frac{d^2y}{dx} = 0$ at $x = 2$ (PT. OF INFLEXION)

$\frac{d^2y}{dx^2} < 0$ at $x = 0$ MAXIMUM; $\frac{d^2y}{dx^2} > 0$ at $x = 4$ MINIMUM.

$\{x = 0 \quad 6x - 12 = -12\}$

$\{x = 4 \quad 6x - 12 = 12\}$

(ii) find Normal to curve at $(2, -4)$

at $x = 2$ $\frac{dy}{dx} = -12$; grad. of tangent at $(2, -4)$ is -12

$\therefore$ gradient of normal $= \frac{1}{12}$ {using $mm' = -1$}

Hence equation of NORMAL is ...

$$(y - (-4)) = \frac{1}{12}(x - 2)$$

$$y = \frac{x}{12} - \frac{1}{6} - 4 \Rightarrow \underline{y = \frac{x}{12} - 4\frac{1}{6}}$$

(11) Table given ... hot drink left to cool.

(x) Time (mins)	10	20	30	40	50
(y) Temp °C	68	53	42	36	31

Room temp°c is 22°C

Difference between temp. of drink and room at t mins is z°c.

modelled by

$$z = z_0 \, 10^{-kt}$$

(i) give an interpretation of z_0 ; EXCESS TEMP° at $t=0$.

(ii) show $\log_{10} z = -kt + \log_{10} z_0$

$$z = z_0 \, 10^{-kt}$$

take logs $\log_{10} z = \log_{10}\left(z_0 \, 10^{-kt}\right)$

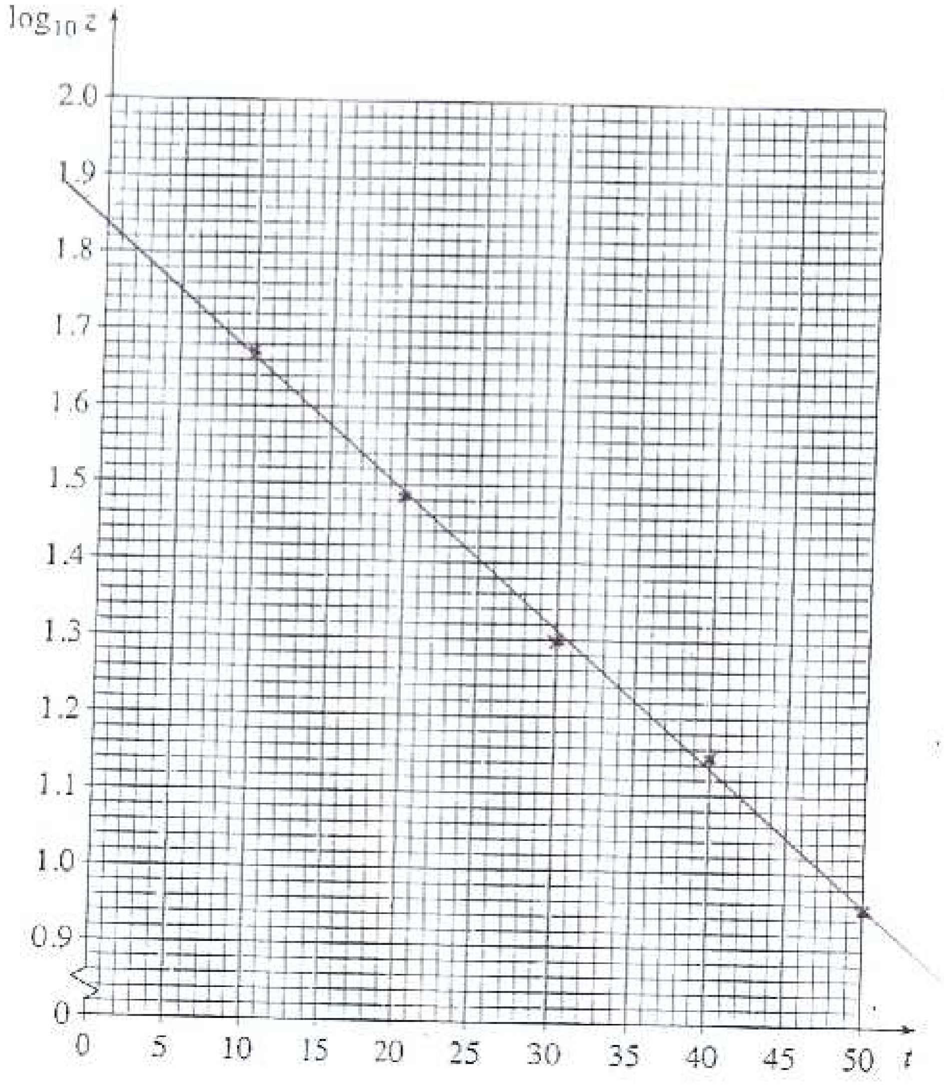
$$= \log_{10} z_0 + \left\{\log_{10} 10^{-kt}\right\} \rightarrow -kt \log_{10} 10 = -kt$$

$$\Rightarrow \log_{10} z = -kt + \log_{10} z_0 \qquad \text{(as required).}$$

See insert ...

11 (iii)

t	10	20	30	40	50
z	46	31	20	14	9
$\log_{10} z$	1.663	1.491	1.301	1.146	0.954



$\log_{10} z_0 = 1.84 \quad \therefore z_0 \approx 64.2 \quad (= 10^{1.84})$

$k = \dfrac{1.663 - 0.954}{46 - 9} \approx 0.019$

when $t = 70$ $\quad z = z_0 10^{-kt} = (69.2) \times 10^{-1.341} \approx 3.24$

hence temp. of drink $= 22 + 3.24 = \underline{\underline{25.24}}\ ^\circ C$