C2 June 2010 — Sect A

① $u_1 = 1 \quad u_{n+1} = \dfrac{u_n}{1+u_n}$

$u_1 = 1$

$u_2 = \frac{1}{2}$

$u_3 = \dfrac{1/2}{1+1/2} = \dfrac{1/2}{3/2} = \dfrac{1}{3}$

$u_4 = \dfrac{1/3}{1+1/3} = \dfrac{1/3}{4/3} = \dfrac{1}{4}.$

② (i) Evaluate $\displaystyle\sum_{r=2}^{5} \frac{1}{r-1} = \frac{1}{2-1} + \frac{1}{3-1} + \frac{1}{4-1} + \frac{1}{5-1}$

$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$

$= 2\frac{1}{12}$

(ii) Express $2\times3 + 3\times4 + 4\times5 + \ldots + 6\times7$

$$= \sum_{r=2}^{6} r(r+1) \quad \left(\sum_{r=2}^{6} (r^2+r) \right)$$

③ differentiate $x^3 - 6x^2 - 15x + 50$

(i) $\rightarrow 3x^2 - 12x - 15 = 3(x^2 - 4x - 5)$

{factorize} (ii) $\dfrac{dy}{dx} = 3(x-5)(x+1) \qquad \dfrac{dy}{dx} = 0$

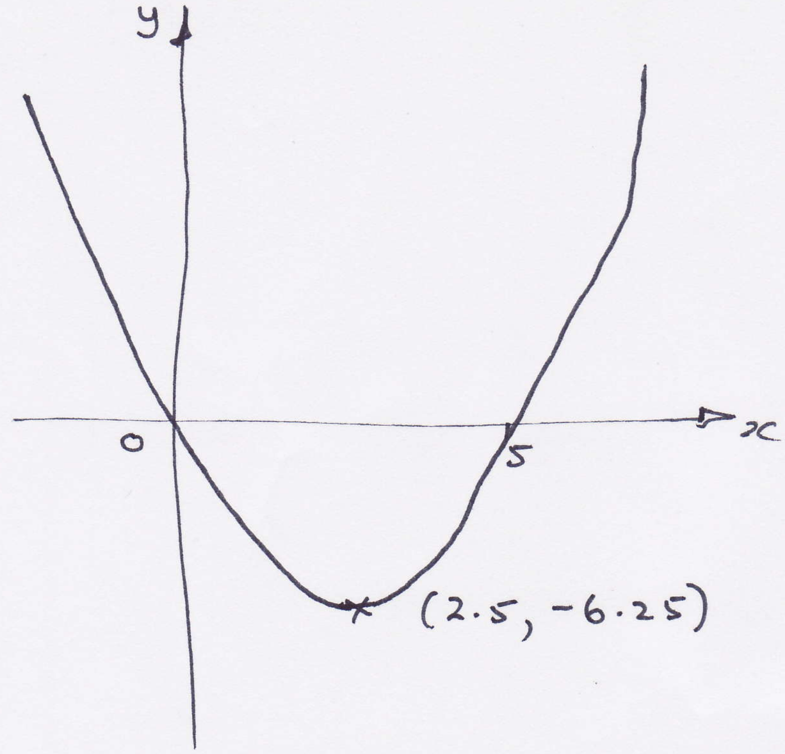
when $x - 5 = 0 \quad x = 5$

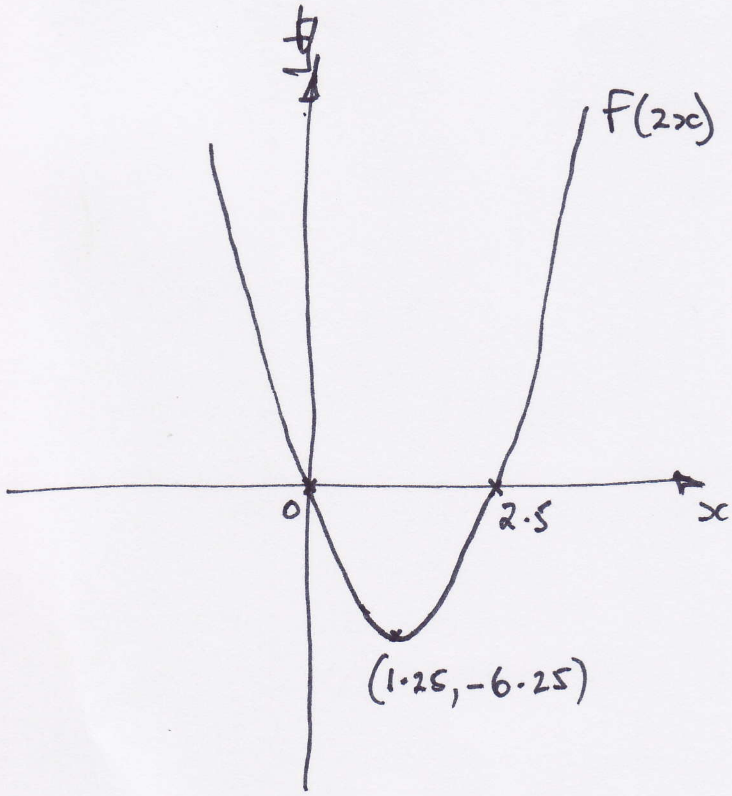
$x + 1 = 0 \quad x = -1$

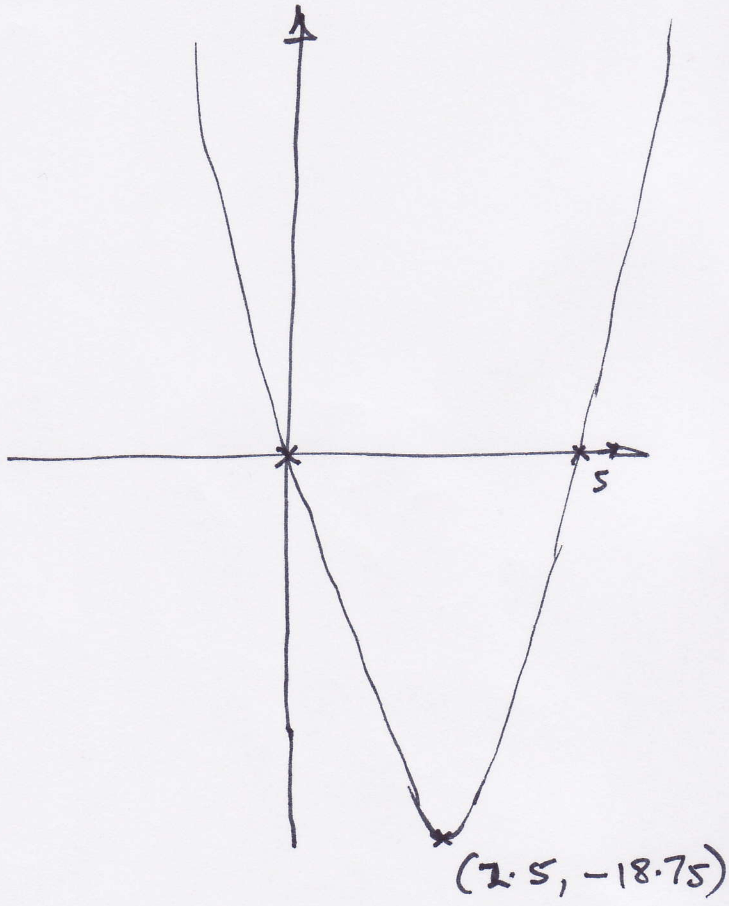
x coordinates of stationary points are -1 and 5.

4

$f(x) = x^2 - 5x$







Apologies for the standard of my sketches!!

(5) $$\int_2^5 \left(1 - \frac{6}{x^3}\right)dx = \int_2^5 \left(1 - 6x^{-3}\right)dx = \Big[\, x + 3x^{-2} \,\Big]_2^5$$

x $\qquad \dfrac{-6x^{-2}}{-2}$

$$= \left(5 + \frac{3}{25}\right) - \left(2 + \frac{3}{4}\right) = \underline{\underline{2.37}}$$

(6) gradient eq^n given as $6x^2 - 12x^{1/2}$

$\Rightarrow \dfrac{dy}{dx} = 6x^2 - 12x^{1/2}$; integrating gives $y = 2x^3 - \dfrac{12x^{3/2}}{3/2} + c$

$\therefore \underline{\underline{y = 2x^3 - 8x^{3/2} + c}}$

curve passes thro' $(4, 10)$

Subst. $\rightarrow$ $10 = 2(4)^3 - 8(4)^{3/2} + c$

$10 = 128 - 64 + c \quad \therefore c = -54$

hence eq^n is $\underline{\underline{y = 2x^3 - 8x^{3/2} - 54}}$

(7) $\log_a x^3 + \log_a \sqrt{x} = 3\log_a x + \frac{1}{2}\log_a x$

$= \underline{\underline{3\tfrac{1}{2}\log_a x}}$

(8) $4\sin^2\theta = 3 + \cos^2\theta$

(using $\sin^2\theta + \cos^2\theta = 1$
and so $\cos^2\theta = 1 - \sin^2\theta$

$4\sin^2\theta = 3 + (1 - \sin^2\theta)$

$\Rightarrow 5\sin^2\theta = 4$

$\sin^2\theta = 4/5$

$\sin\theta = \sqrt{4/5}$

$\theta = 63.434\ldots^\circ$

in the range $0^\circ \rightarrow 360^\circ$

$\theta = 180 - 63.434^\circ$

$\therefore \theta = \underline{63.4 \text{ and } 116.6^\circ}$ (1dp)

or $\sin^2\theta = 1 - \cos^2\theta$

ALTERNATIVELY

$4(1-\cos^2\theta) = 3 + \cos^2\theta$

$1 = 5\cos^2\theta$

So $\cos^2\theta = \frac{1}{5}$

$\cos\theta = \sqrt{1/5}$

$\theta = 63.434\ldots^\circ$

(but in this case..

$\theta = 360 - 63.434\ldots$

So $\theta = 63.4^\circ$
and 296.6°

BUT BEWARE . . .

$\sin\theta = \pm\sqrt{4/5}$ OR $\cos\theta = \pm\sqrt{\frac{1}{5}}$

this will yield values ...

63.4°, 116.6°, $\underline{243.4^\circ}$ and 296.6°

A BIT OF A BEAST ! ! !

(9) $(2,6)$ and $(3,18)$ lie on $y = ax^n$

So $6 = a \times 2^n$ and $18 = a \times 3^n$ } taking logs $\log 6 = \log(a \times 2^n)$, $\log 18 = \log(a \times 3^n)$

hence $\log 6 = \log a + n \log 2$ —— (A)

and $\log 18 = \log a + n \log 3$ —— (B)

eqn (B) − eqn (A): $\log 18 - \log 6 = (\cancel{\log a} + n\log 3) - (\cancel{\log a} + n\log 2)$

cancelling, and taking n as a common factor → $= n(\log 3 - \log 2)$

gives

$$\log 18 - \log 6 = n(\log 3 - \log 2)$$

$$\therefore n = \frac{\log 18 - \log 6}{\log 3 - \log 2} \approx \underline{\underline{2.71}}$$

Substitute the full display for n gives...

~~and~~ $\log 6 = \log a + (2.7095\ldots)\log 2$

$\Rightarrow \log a = \log 6 - (2.7095\ldots)\log 2$

$\log a = -0.0374\ldots$

$\Rightarrow a = 10^{-0.0374\ldots} = 0.9172\ldots$

$$\underline{\underline{a = 0.92}}$$

NOTE: There is an alternative method!!

but the answers are the same.

R.M.S.