C2 MAY 2005

① Differentiate $x + \sqrt{x^3}$

$$x + \sqrt{x^3} = x + (x^3)^{1/2} = x + x^{3/2}$$

$$\therefore \frac{dy}{dx} = 1 + \frac{3}{2}x^{1/2} \quad \underline{\underline{\left(= 1 + \frac{3\sqrt{x}}{2} \right)}}$$

② nth term of an A.P. is $6+5n$

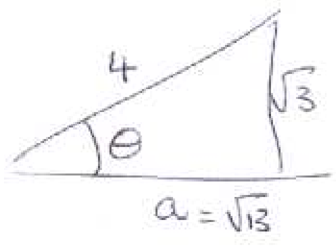
so $t_1 = 6+5 = 11$

$t_{20} = 6 + 5 \times 20 = 106$

Sum to 20 terms $= \frac{1}{2} \times 20 \left(11 + 106 \right) = 10 \times 117 = \underline{\underline{1170}}$

$\left\{ \text{using sum} = \frac{1}{2} n (a + l) \text{ where } a = \text{1st term } l = \text{last term} \right\}$

③ $\sin\theta = \frac{\sqrt{3}}{4} \Rightarrow$



$$a = \sqrt{4^2 - (\sqrt{3})^2} = \sqrt{16-3} = \sqrt{13}$$

$$\therefore \cos\theta = \underline{\underline{\frac{\sqrt{13}}{4}}}$$

④ $y = x + \frac{1}{x} = x + x^{-1}$ $\quad \therefore \frac{dy}{dx} = 1 - x^{-2} = 1 - \frac{1}{x^2}$

$x = 1 \quad \frac{dy}{dx} = 1 - \frac{1}{1} = 0 \quad \therefore$ turning pt.

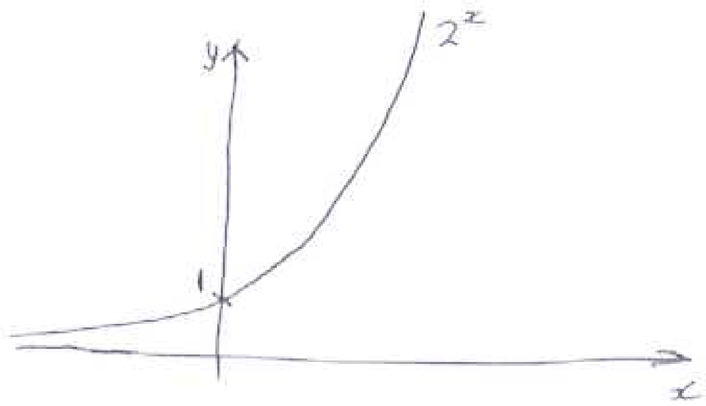
using $\frac{d^2y}{dx^2} = 2x^{-3} = \frac{2}{x^3}$ $\quad$ when $x=1$ $\frac{d^2y}{dx^2} = 2$ $\quad \frac{d^2y}{dx^2} > 0$

$\therefore$ <u>MINIMUM</u>

⑤ (i) $\log_5 5 = 1$ (ii) $\log_3\left(\frac{1}{9}\right) = \log_3 3^{-2} = \underline{\underline{-2}}$.

(iii) $\log_a x + \log_a x^5 = \log_a x + 5\log_a x = \underline{\underline{6\log_a x}}$

⑥ sketch $y = 2^x$



if $2^x = 50$ taking natural logs

$\ln 2^x = \ln 50$

$x \ln 2 = \ln 50$

$x = \dfrac{\ln 50}{\ln 2} \approx 5.64.$

OR
$\log_{10} 2^x = \log_{10} 50$

$x\log_{10} 2 = \log_{10} 50$

$x = \dfrac{\log_{10} 50}{\log_{10} 2} \approx 5.64$

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⑦ gradient of curve given by $\dfrac{dy}{dx} = \dfrac{6}{x^3}$; passes thro' $(1,4)$

Find eqn of curve.

integrating $\dfrac{6}{x^3}$ $\left(\int 6x^{-3}\right)$

gives $\dfrac{6x^{-2}}{-2} + c = \dfrac{-3}{x^2} + c$ but $y = -\dfrac{3}{x^2} + c$ satisfied by $(1,4)$

$\therefore 4 = -\dfrac{3}{1} + c \quad c = 7$

hence curve is $\underline{\underline{y = -\dfrac{3}{x^2} + 7}}$

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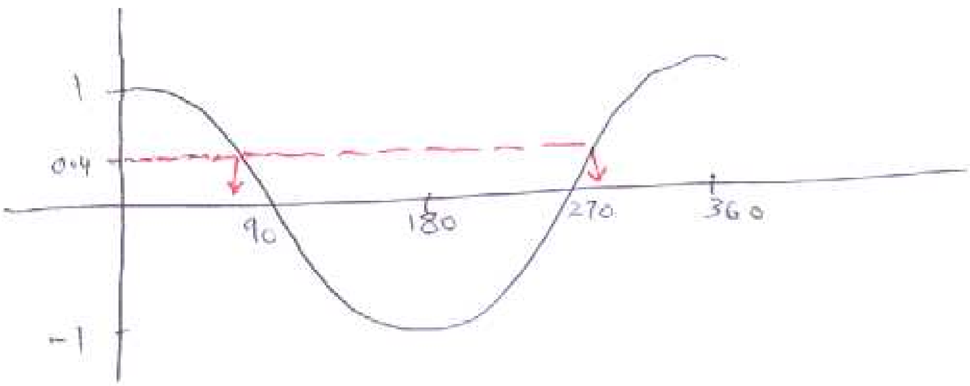
(i) Solve $\cos\theta = 0.4 \quad 0 \le x \le 360^\circ$



$\cos\theta = 0.4 \qquad \theta = \cos^{-1} 0.4 = 66.42^\circ$

using the graph $\theta = 66.42^\circ$ and $360^\circ - 66.42^\circ$

$= 66.42^\circ$ and 293.58°

$y = \cos x \rightarrow y = \cos 2x$

Stretch by factor $\frac{1}{2}$ in x dir^n

(scaling in x dir^n)