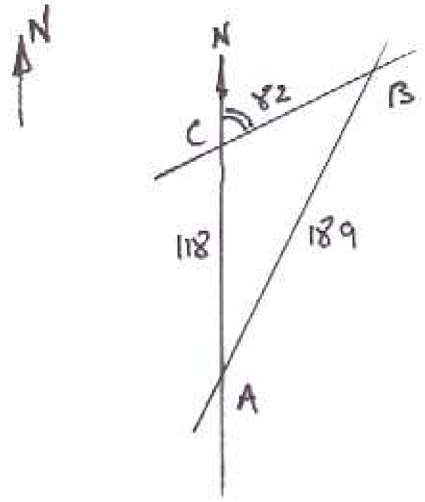
C2 - JAN 2007 - SECT B.

11.



(i) Calc. bearing of B from C (find ∡ marked).

using Cos Rule $\cos C = \dfrac{82^2 + 118^2 - 189^2}{2 \times 82 \times 118} = -0.7788\ldots$

{you should show full calc. display ... I haven't in this solution!)

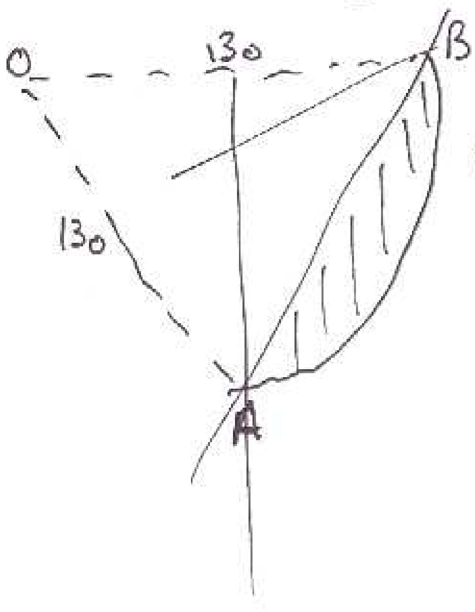
$\therefore C = \cos^{-1}(-0.7788\ldots) \approx 141^\circ$ (To nearest degree)

$\therefore$ bearing of B from C: $180 - 141 = \underline{\underline{039^\circ}}$

{must give 3 fig. bearing}

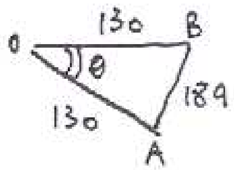
(ii) using $C = 141$ and $\frac{1}{2}ab\sin C$ gives...

Area $= \dfrac{1}{2} \times 82 \times 118 \times \sin 141^\circ = \underline{\underline{3034}}\ \text{m}^2$



arc is new road

pt.(A)



$A\hat{O}B \rightarrow$ IN RADIANS (take care on your calculator!!)

$\cos\theta = \dfrac{130^2 + 130^2 - 189^2}{2 \times 130 \times 130} = -0.05683\ldots$

$\theta = \cos^{-1}(-0.05683\ldots) = 1.62766\ldots$ rads

$= \underline{\underline{1.63 \text{ rads}}}$.

11 (contd.)

pt. (B) Shaded area = Sector AOB $-$ Δ AOB

$$= \frac{1}{2} r^2 \theta - \frac{1}{2} \times 130 \times 130 \times \sin 1.63$$

in rads.

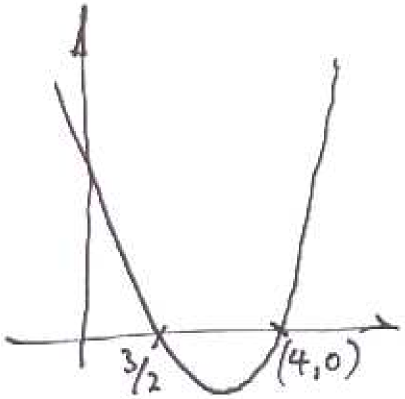
$$= \frac{1}{2} \times 130^2 \times 1.63 - \frac{1}{2} \times 130^2 \times \sin 1.63$$

$$= 13753.73\ldots - 8435.195\ldots$$

$$= 5318.53\ldots$$

$$= \underline{\underline{5300 \text{ m}^2}} \quad (2 \text{ S.F.})$$

(12)



$$y = 2x^2 - 11x + 12$$

(i) Solving $2x^2 - 11x + 12 = 0$

$$(2x - 3)(x - 4) = 0$$

$$\therefore x = 4 \quad \text{and} \quad x = 3/2$$

hence points of intersection are $(4, 0)$ and $(3/2, 0)$

(ii) Find eqn of normal at $(4, 0)$

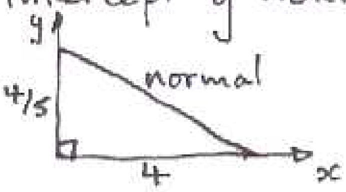
$\frac{dy}{dx} = 4x - 11$ $\therefore$ gradient of tangent at $(4,0)$ is $4 \times 4 - 11 = 5$.

using $mm^{-1} = -1$ gradient of normal is $-1/5$

So eqn of NORMAL at $(4,0)$ $y - 0 = -1/5(x - 4)$

$$y = 4/5 - x/5 \quad [x + 5y = 4]$$

intercept of NORMAL is $(0, 4/5)$

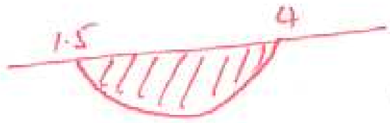


AREA of $\Delta = \frac{1}{2} \times 4 \times \frac{4}{5} = \frac{8}{5} = \underline{\underline{1.6 \text{ units}^2}}$

(12) (iii) Area of region bounded by curve and the x axis .....

{an ambiguous and misleading question .... as no limits are given}.

THE MARK SCHEME infers that the limits are $1.5 < x < 4$ hence



.....

$$y = 2x^2 - 11x + 12$$

$$\int y \, dx = \frac{2x^3}{3} - \frac{11x^2}{2} + 12x \quad (+c)$$

Solving with limits $1.5 < x < 4$

gives $\left[\frac{2x^3}{3} - \frac{11x^2}{2} + 12x\right]_{1.5}^{4}$

$$= \left(\frac{2 \times 64}{3} - \frac{11 \times 16}{2} + 48\right) - \left(\frac{2(1.5)^3}{3} - \frac{11(1.5)^2}{2} + 18\right)$$

$$= \left(\frac{128}{3} - 88 + 48\right) - (2.25 - 12.375 + 18)$$

$$= 2.66\ldots - 7.875 = -5.2083\ldots$$

$$= -5.21 \quad (3.S.F.)$$

{note: $-$ sign shows the AREA is below x axis!}

(13) Table

x	1	2	3	4	5	6
y	500	800	1200	1900	3000	4800

firm's profit, £y for the xth month after start-up is given by

$$y = k \times 10^{ax}$$

(i)

$$\log_{10} y = \log_{10}\left(k \times 10^{ax}\right)$$

$$= \log_{10} k + \log_{10} 10^{ax}$$

$$= \log_{10} k + ax$$

hence $\log_{10} y = ax + \log_{10} k$ is a straight line

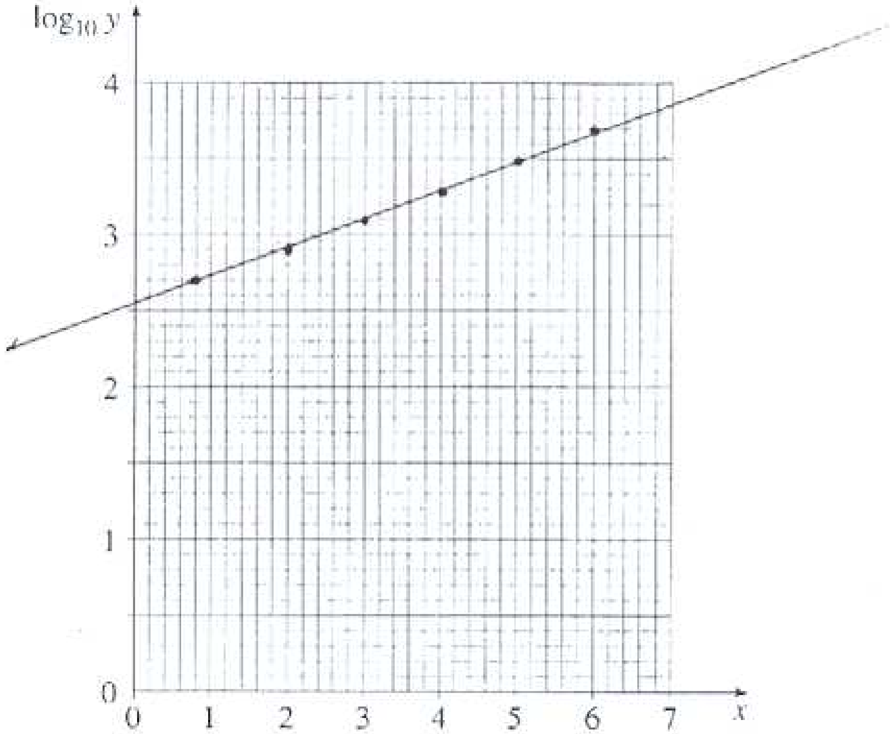
(eqn of $y = mx + c$)

with gradient a, and intercept $\log_{10} k$

(ii) SEE INSERT

13 (ii)

Number of months after start-up (x)	1	2	3	4	5	6
Profit for this month (£y)	500	800	1200	1900	3000	4800
$\log_{10} y$	2.70	2.90	3.08	3.28	3.48	3.68



(iii)

from graph ... intercept 2.55

$$\text{gradient} = \frac{3.68 - 3.28}{6 - 4} = \frac{0.4}{2} = 0.2$$

hence $\log_{10} y = 0.2x + 2.55$

$$\underline{\underline{y = 10^{(0.2x + 2.55)}}}$$

(iv)

$$75000 = 10^{(0.2x + 2.55)}$$

$$75000 = 10^{2.55} \times 10^{0.2x} \approx 355 \times 10^{0.2x}$$

$$\therefore 10^{0.2x} = \frac{75000}{355} \approx 211 \implies 0.2x = \log_{10} 211$$

$$0.2x = 2.325$$

(v) PROFITS VARY !!

$$\underline{\underline{x = 11.62\ldots}}$$