C2 JUNE '06 – SECT A.

① $\log_a a = 1$ $\quad \log_a a^3 = 3\log_a a = 3\times 1 = 3.$

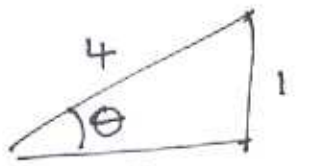
② G.P. → $a, ar, ar^2, ar^3, \ldots$

given $a = 8$ $\quad S_\infty = 10$

$\therefore \quad 10 = \dfrac{a}{1-r} = \dfrac{8}{1-r} \Rightarrow 1-r = \dfrac{8}{10}$

$\therefore r = \dfrac{2}{10} = 0.2 \quad \left(\frac{1}{5}\right)$

③ θ is acute $\sin\theta = \frac{1}{4} \Rightarrow$



$a = \sqrt{4^2-1^2} = \sqrt{15}$

$\therefore \tan\theta = \dfrac{1}{\sqrt{15}}$

④ $\displaystyle\int_1^2 \left(x^4 - \frac{3}{x^2} + 1\right)dx = \int_1^2 \left(x^4 - 3x^{-2} + 1\right)dx$

$\displaystyle = \left[\frac{x^5}{5} - \frac{3x^{-1}}{-1} + x\right]_1^2$

$\displaystyle = \left[\frac{x^5}{5} + \frac{3}{x} + x\right]_1^2$

$= \left(\frac{2^5}{5} + \frac{3}{2} + 2\right) - \left(\frac{1}{5} + 3 + 1\right)$

$= 9\frac{9}{10} - 4\frac{2}{10} = 5\frac{7}{10} \quad (5.7)$

(5)

gradient of curve given by $\frac{dy}{dx} = 3 - x^2$

curve passes thro' (6, 1)

$$\int (3 - x^2)\, dx = 3x - \frac{x^3}{3} + c$$

but $y = 3x - \frac{x^3}{3} + c$ satisfied by (6, 1)

$$1 = 3 \times 6 - \frac{6^3}{3} + c \Rightarrow c = 1 + 72 - 18$$
$$= 55$$

eqn of curve is

$$y = 55 + 3x - \frac{x^3}{3}$$

(6) sequence ... $u_1 = 3$

$$u_{n+1} = u_n + 5$$

(i) $\Rightarrow$ 3, 8, 13, 18 an A.P. with $a = 3$, $d = 5$

(ii) find Sum of 51st to 100th term ...

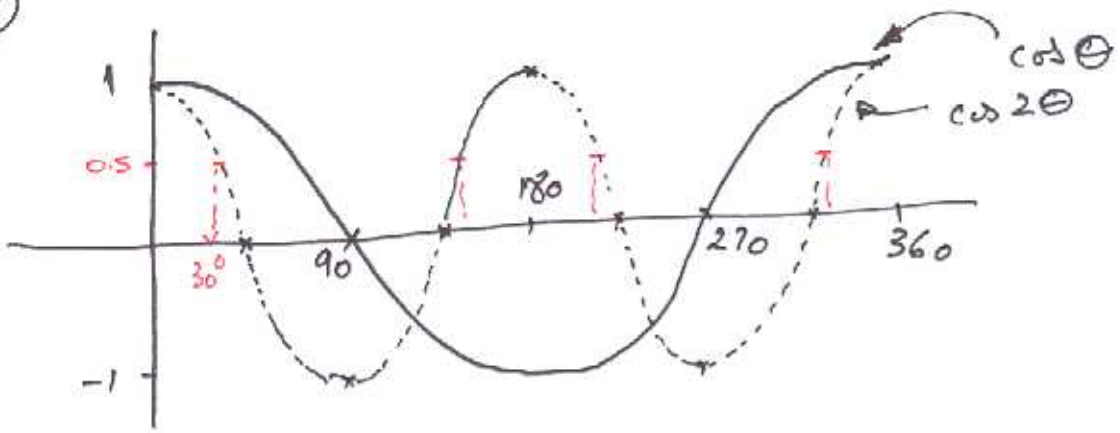
$$u_{51} = 3 + 50 \times 5 = 253$$
$$u_{100} = 3 + 99 \times 5 = 498$$

$\therefore$ Sum $\rightarrow \sum_{51}^{100} \rightarrow \frac{1}{2} n (a + l)$ where $a = 253$, $l = 498$, $n = 50$

$$\frac{1}{2} \times 50 (253 + 498)$$

$$= 18775$$

(7)



(ii) Solve $\cos 2x = 0.5$

$2x = \cos^{-1}(0.5) = 60°$

$\therefore x = 30°$

$\therefore x = 30°, (180-30)°, (180+30)°, (360-30)°$

$= 30°, 150°, 210°, 330°$

(8)

$y = 6x^3 + \sqrt{x} + 3 \quad (= 6x^3 + x^{1/2} + 3)$

$\therefore \frac{dy}{dx} = 18x^2 + \frac{1}{2}x^{-1/2}$

$\frac{d^2y}{dx^2} = 36x - \frac{1}{4}x^{-3/2}$

(9)

$5^{3x} = 100 \qquad$ (take logs)

$\log_{10} 5^{3x} = \log_{10} 100$

$3x \log_{10} 5 = 2$

$x = \frac{2}{3\log_{10} 5} = 0.9537\ldots \approx 0.954$ (3.d.p)