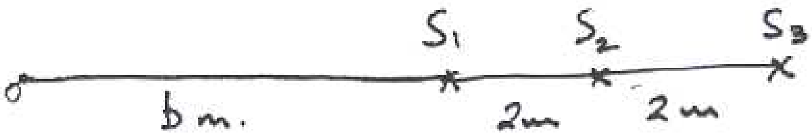
C2 SPECIMEN – SECT B.

No. 9.

"A shuttle $O \to S_1$ & back and so on"



(i) Show total dist. with 3 skittles is $6(b+2)$ metres.

$$\text{Total dist} = 2b + 2(b+2) + 2(b+4)$$

$$= 6b + 4 + 8 = 6b + 12 = \underline{\underline{6(b+2)}}$$

(ii) Show total distance with n skittles is $2n(b+n-1)$.

$$\text{Sum to n terms} = \frac{1}{2}n(a+l)$$

a (first term) = $\quad$ l (last term) = $2b + 4(n-1)$

$$\therefore \text{Sum} = \frac{1}{2}n\left(2b + (2b + 4(n-1))\right)$$

$$= \frac{n}{2}(2b + 2b + 4n - 4) = \frac{n}{2}(4b + 4n - 4)$$

$$= \frac{4n}{2}(b + n - 1)$$

$$= \underline{\underline{2n(b+n-1)}} \text{ as reqd.}$$

OR using $\frac{1}{2}n(2a + (n-1)d)$ with $a = 2b$ $d = 4$

$$\frac{n}{2}(4b + 4(n-1)) = 2n(b+n-1)$$

(as above)

(iii) with $b = 5$ distance $= 570$

find no. of skittles

$$2n(5+n-1) = 570$$

$$2n^2 + 8n - 570 = 0$$

$$n^2 + 4n - 285 = 0$$

$$(n-15)(n+19) = 0 \Rightarrow n = 15, (-19)$$

$\Rightarrow$ 15 skittles.

(iv)

$$2n(b+n-1) = 1000$$

$$n(b+n-1) = 500$$

$b < 20$ $(b \subseteq \mathbb{N})$.

(n is a factor of 500).

$$b+n-1 = \frac{500}{n}$$

only 25 works, giving $b = 16$.

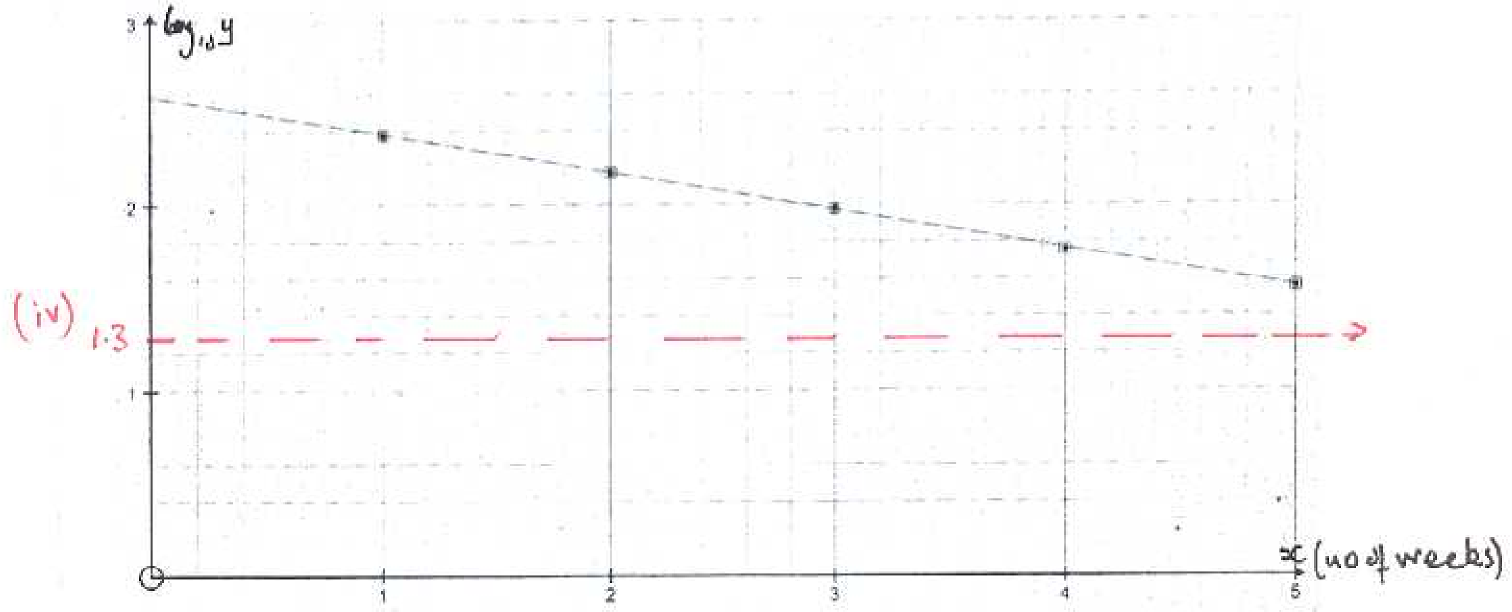
10.

wk. no. x	1	2	3	4	5
no. of new cases y	240	150	95	58	38

no. of new cases, y, in week x is given by $y = pq^x$

x	1	2	3	4	5
$\log_{10} y$	2.3802	2.1761	1.9777	1.7634	1.5798

plotting the graph gives a straight line.



$y = pq^x$ take logs $\log_{10} y = \log_{10} pq^x$

$\log_{10} y = \log_{10} p + \log_{10} q^x$

$\log_{10} y = \log_{10} p + x \log_{10} q$

$\log_{10} y$ against x gives a straight line!

$\log_{10} y = \log_{10} p + x \log_{10} q$

$Y = C + x M \Rightarrow y = mx + c$

10 (contd.)

(iv) using logs $\qquad \log_{10} 20 = 1.30 \qquad$ (2 d.p)

(see graph)

week 7 is outside the x range given

CAUTION ... you are having to use EXTRAPOLATION!

(v) $\quad$ as $\quad \log_{10} y = \underbrace{\log_{10} q}_{\text{gradient.}}\, x + \log_{10} p$

$$\log_{10} q = \underbrace{\frac{1.58 - 2.38}{5 - 1}}_{\text{def}^{n} \text{ of gradient } \frac{\text{inc in } y}{\text{inc. in } x}} = \frac{-0.8}{4} = -0.2 \quad \left(-\tfrac{1}{5}\right)$$

$\log_{10} q = -0.2$

$10^{\log_{10} q} = 10^{-0.2}$

$\therefore q = 0.63$

intercept is $\log_{10} p = 2.58 \Rightarrow p = 380$

using $y = pq^{x}$ with $x = 3$ $\quad \therefore y = 380 \times 0.63^{3} = 95.0$

(Agrees with data).

11. $y = x^4 - 8x^2 + 7$

(i) $\dfrac{dy}{dx} = 4x^3 - 16x = 4x(x^2 - 4)$

$= 4x(x+2)(x-2)$

$\dfrac{dy}{dx} = 0$ when $x = 0, -2, +2.$

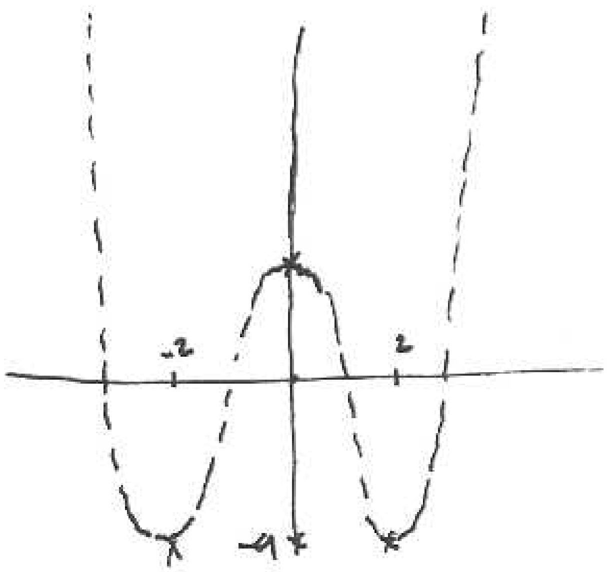
when $x = 2$ $y = 2^4 - 8(2)^2 + 7 = 16 - 32 + 7 = -9$

at $\underline{(2, -9)}$ $\dfrac{dy}{dx} = 0$ also at $\underline{(0, 7)}$

and when $x = -2$ $y = (-2)^4 - 8(-2)^2 + 7$

$= 16 - 32 + 7 = -9.$

and $\underline{(-2, -9)}$



(iii) $y = -12x + 12$

$y = -12(x - 1)$

pt. $(1, 0)$ satisfies tangent equation

and at $x = 1$ $\dfrac{dy}{dx} = 4 - 16 = -12$

$(y = x^4 - 8x^2 + 7)$

$y = 0$ $0 = x^4 - 8x^2 + 7$

with $x = 1$ $1 - 8 + 7 = 0$