C2 JAN 2007 — SECT. A.

No. 1. $y = 6x^{5/2} + 4 \quad \therefore \quad \dfrac{dy}{dx} = 6 \times \dfrac{5}{2} x^{3/2}$

$$\frac{dy}{dx} = 15x^{3/2}$$

(2) G.P with $a = 6$ $\quad S_\infty = 5$ $\quad$ find r.

$$S_\infty = \frac{a}{1-r} \Rightarrow 5 = \frac{6}{1-r} \Rightarrow 1 - r = \frac{6}{5}$$

re-arranging $\quad \underline{r = 1 - 6/5 = -1/5}$

(3) $\cos\theta = 1/3$ (acute)

$\sqrt{3^2 - 1^2} = \sqrt{8}$

(use pythagoras

from Δ $\sin\theta = \dfrac{\sqrt{8}}{3}$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \rightarrow \frac{\sqrt{8}}{3} \div \frac{1}{3} = \frac{\sqrt{8}}{\cancel{3}} \times \frac{\cancel{3}}{1} = \sqrt{8}$$

$$\underline{\tan\theta = \sqrt{8}}$$

(4) (i) C PERIODIC

(ii) B CONVERGENT.

(iii) nth term of A $\rightarrow 2^{n-1}$

A: 1, 2, 4, 8, 16, 32

B: 20, −10, 5, −2.5, 1.25, −0.625, ...

C: 20, 5, 1, 20, 5, 1, ...

⑤ $A(2,1)$ is on the curve $y = \frac{4}{x^2}$

pt. B is on same curve with x coord. 2.1

(i) gradient of AB: $y_B = \frac{4}{2.1^2} = \frac{4}{4.41}$

$$\text{grad. of } AB = \frac{\frac{4}{4.41} - 1}{2.1 - 2} = \frac{-0.09297\ldots}{0.1}$$

$$= -0.92970\ldots$$

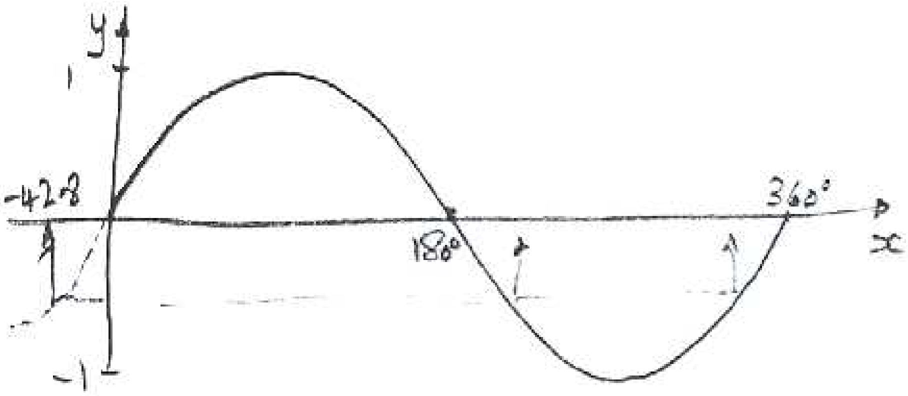
$$\approx \underline{-0.93}$$

(ii) any such that $2 < x < 2.1$

(iii) $y = \frac{4}{x^2} \rightarrow y = 4x^{-2}$ $\quad \frac{dy}{dx} = -8x^{-3} = -\frac{8}{x^3}$

at A, when $x=2$ $\quad \frac{dy}{dx} = -\frac{8}{2^3} = -1$

⑥



$\sin x = -0.68$

$x = \sin^{-1}(-0.68) = -42.8^\circ$ (1 d.p.)

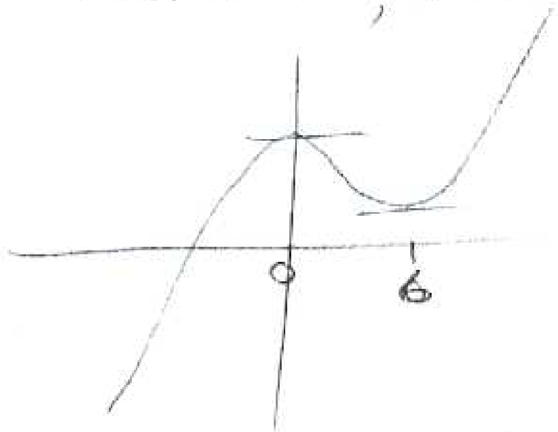
$\therefore x = 180 + 42.8$

and $360 - 42.8$

ans. $\underline{222.8,\ 317.2}$

(7) $\frac{dy}{dx} = x^2 - 6x \quad (= x(x-6))$

$\frac{dy}{dx} = 0$ when $x = 0$, $x = 6$



$\frac{dy}{dx}$ is increasing $\left(\frac{dy}{dx} > 0\right)$

$\underline{x < 0, \quad x > 6.}$

OR SIGN DIAG. for dy/dx

x: $- - - - \; | \; + + + + +$ (0)

$x - 6$: $- - - - - - - \; | \; + +$ (6)

$x(x-6)$: $+ + + + \; | \; - - - \; | \; + +$ (0, 6)

(8) t_7 of A.P. is 6. Sum to 10 terms is 30.

find 5th term.

$a + 6d = 6$ and $5(2a + 9d) = 30$

Solving
$$a + 6d = 6 \quad \text{— A}$$
$$2a + 9d = 6 \quad \text{— B}$$

$$2a + 12d = 12$$
$$2a + 9d = 6$$
$$3d = 6 \quad \underline{d = 2} \quad \therefore a = -6.$$

$t_5 = -6 + 4 \times 2 = \underline{\underline{2}}$

$t_5 = a + (n-1)d$
$= -6 + 4 \times 2$

(9) gradient of curve given by

$$\frac{dy}{dx} = 6x^2 + 8x \quad , \text{ passes thro' pt. } (1,5)$$

find eqn of curve

Integrating gives $y = 6\frac{x^3}{3} + \frac{8x^2}{2} + c$

$$y = 2x^3 + 4x^2 + c$$

but when $x = 1$ $y = 5$

$$5 = 2(1)^3 + 4(1)^2 + c \quad \therefore c = -1$$

eqn of curve

$$\underline{\underline{y = 2x^3 + 4x^2 - 1}}$$

10. (i) $\log_a x^4 + \log_a\left(\frac{1}{x}\right) = 4\log_a x + \log_a x^{-1}$

$$= 4\log_a x - \log_a x$$

$$\underline{\underline{= 3\log_a x}}$$

(ii) $\log_{10} b + \log_{10} c = 3$ (take exponents)

$$10^{(\log_{10} b + \log_{10} c)} = 10^3$$

$$10^{\log_{10} bc} = 10^3$$

$$bc = 1000 \implies b = \frac{1000}{c}$$