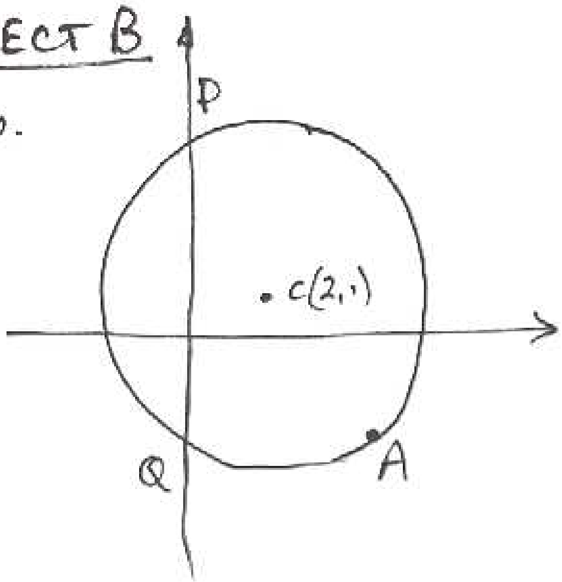
SECT B JAN 2005

10.



given centre $(2,1)$
radius 5

using $(h,k) = (2,1)$ and $r = 5$

eqⁿ of circle is $(x-h)^2 + (y-k)^2 = r^2$

$$\Rightarrow (x-2)^2 + (y-1)^2 = 25$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 - 25 = 0$$

$$\underline{x^2 + y^2 - 4x - 2y - 20 = 0} \quad \text{(as reqd.)}$$

cuts the y axis when $x = 0$

So subst. $x = 0$ into circle eqn. gives

$$y^2 - 2y - 20 = 0$$

won't factorise; so use 'formula' OR 'completed square'

COMPLETING SQUARE

$$(y-1)^2 - 21 = 0$$

$$(y-1)^2 = 21$$

$$y - 1 = \pm\sqrt{21}$$

$$\underline{y = 1 \pm \sqrt{21}}$$

FORMULA

$a = 1 \quad b = -2 \quad c = -20$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-20)}}{2}$$

$$= \frac{2 \pm \sqrt{4 + 80}}{2}$$

$$= \frac{\cancel{2}^{1}}{\cancel{2}} \pm \frac{\sqrt{84}}{2}$$

$\sqrt{84} = \sqrt{4 \times 21}$

$$= 1 \pm \frac{\cancel{2}\sqrt{21}}{\cancel{2}} = \underline{1 \pm \sqrt{21}}$$

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10(iii)

A(5, −3) subst. into eqⁿ of circle

$5^2 + (-3)^2 - 4(5) - 2(-3) - 20$

$= 25 + 9 - 20 + 6 - 20 = \underline{\underline{0}}$.

So (5, −3) lies on Circle !

11. $f(x) = x^3 + x^2 - 10x + 8$

$f(1) = 1^3 + 1^2 - 10 + 8 = 0 \quad \therefore (x-1)$ is a factor

$f(x) = (x-1)(x^2 + px - 8)$

EITHER divide $x^3+x^2-10x+8$ by $(x-1)$ and then solve the quadratic....

OR compare coefficients.....

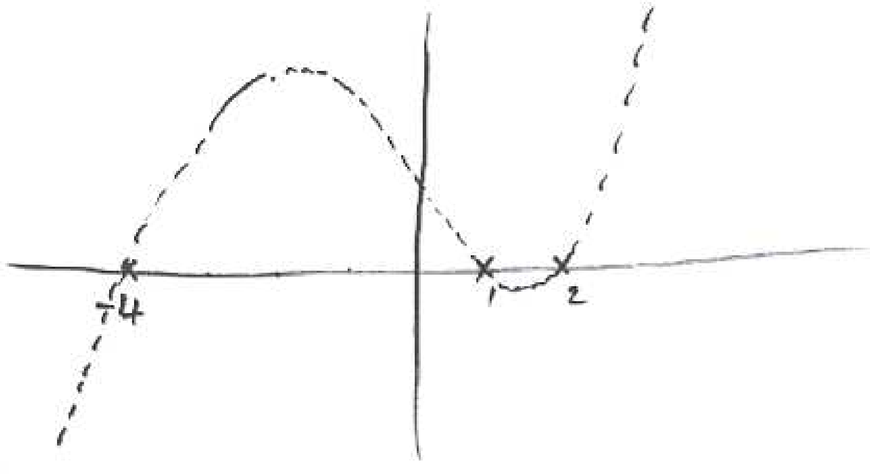
$x^3 + x^2 - 10x + 8 = (x-1)(x^2 + px - 8)$

Coeffs of x L.H.S -10 = R.H.S $-8-p$

$\therefore \underline{\underline{p = 2}}$

$(x-1)(x^2+2x-8) = (x-1)(x-2)(x+4)$

So $f(x) = (x-1)(x-2)(x+4)$



11. (ii)

$f(x)$ is translated by $\begin{pmatrix} -3 \\ 0 \end{pmatrix} \Rightarrow f(x+3)$

hence the eqn of the graph is

$f(x+3) = (x+3)^3 + (x+3)^2 - 10(x+3) + 8$

(NO NEED TO Simplify).

but the INTERCEPT given by the addition of the CONSTANTS

$3^3 + 3^2 - 30 + 8 = 27 + 9 - 30 + 8 = \underline{\underline{14}}$

INTERCEPT is $(0, 14)$

OR as $f(x) = (x-1)(x-2)(x+4)$

$f(x+3) = (x+2)(x+1)(x+7)$

giving CONSTANT of $2 \times 1 \times 7 = \underline{\underline{14}}$

$\therefore$ INTERCEPT is $\underline{\underline{(0, 14)}}$

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No. 12.

(i) $y = x^2 - 3x + 11$

{N.B. for a quadratic graph to be above the x axis for all values, $b^2 - 4ac < 0$}

So... $a = 1$ $b = -3$ $c = 11$

$$b^2 - 4ac = (-3)^2 - 4 \times 1 \times 11$$
$$= 9 - 44 = \underline{-35}$$

as $b^2 - 4ac < 0$ then $y = x^2 - 3x + 11$ is above x axis

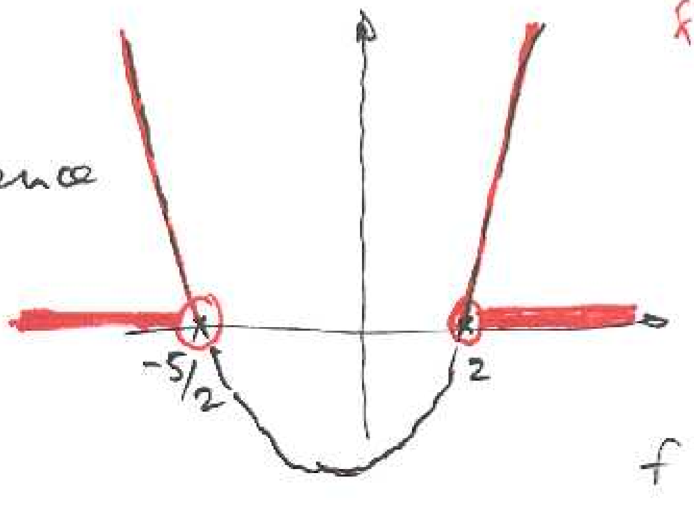
(ii) $y = 2x^2 + x - 10$; solving!! $2x^2 + x - 10 = 0$

factorise $(2x + 5)(x - 2) = 0$

$2x + 5 = 0$ $x = -5/2$

or $x - 2 = 0$ $x = 2$.

Hence



$\therefore$ $2x^2 + x - 10 > 0$ (above the x axis)

f $\underline{\underline{x < -5/2 \text{ and } x > 2}}$

(iii) if $y = x^2 - 3x + 11$ and $y = 2x^2 + x - 10$

then $2x^2 + x - 10 = x^2 - 3x + 11$ $\Rightarrow$ $x^2 + 4x - 21 = 0$

factorise!! $(x + 7)(x - 3) = 0$

$x = -7$ and 3

Subst. $x = -7$ and $x = 3$
into $x^2 - 3x + 11$ gives $y = 81$ and 11 } Ans: $(-7, 81)$ and $\underline{(3, 11)}$