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C1 SECT B.

10. (i) circle ... $x^2 + y^2 = 45$

has CENTRE $(0, 0)$; RADIUS $\sqrt{45}$ $(= 3\sqrt{5})$

(ii) Circle intersects line $x + y = 3$ at 2 points A, B.

Find coordinates of A and B

SIMULTANEOUS
EQUATIONS!

$$x^2 + y^2 = 45 \quad \text{--- (1)}$$

$$x + y = 3 \Rightarrow y = 3 - x \quad \text{--- (2)}$$

Subst. for y in eqn (1)

$$x^2 + (3 - x)^2 = 45$$

$$x^2 + 9 - 6x + x^2 = 45$$

$$2x^2 - 6x - 36 = 0$$

$$2(x^2 - 3x - 18) = 0$$

$$2(x + 3)(x - 6) = 0 \Rightarrow x = -3 \text{ and } 6$$

Subst. $x = -3$ and 6 in (2): $y = 6$ and -3

$\therefore A(-3, 6)$ $B(6, -3)$

now using
PYTHAGORAS!

$$AB = \sqrt{(-3 - 6)^2 + (6 - (-3))^2}$$

$$= \sqrt{81 + 81} = \sqrt{162}$$

(as req'd.)

No. 11. (i) $x^2 - 7x + 6$

(completing the square)

$$(x - 3\frac{1}{2})^2 - (3\frac{1}{2})^2 + 6$$

$$\left[(x - \frac{7}{2})^2 - (\frac{7}{2})^2 + 6 \right]$$

$$= (x - 7/2)^2 - \frac{25}{4}$$

$$\begin{aligned} & -\frac{49}{4} + \frac{24}{4} \\ & = -\frac{25}{4} \end{aligned}$$

or

$$\underline{\underline{(x - 3\frac{1}{2})^2 - 6\frac{1}{4}}}$$

(ii) minimum point $\underline{\underline{(3\frac{1}{2}, -6\frac{1}{4})}}$

(iii) crossing points of $y = x^2 - 7x + 6$ on axes!

let $x = 0$ $0^2 - 7 \times 0 + 6 = 6$ $y = 6$

let $y = 0$ $x^2 - 7x + 6 = 0$

$$(x - 6)(x - 1) = 0 \Rightarrow x = 1 \text{ and } 6.$$

hence crossing pts are $(0, 6)$; $(1, 0)$; $(6, 0)$.

(iv) with $y = x^2 - 7x + 6$

and $y = x^2 - 3x + 4$

$$\cancel{x^2} - 7x + 6 = \cancel{x^2} - 3x + 4 \Rightarrow -4x = -2$$

$$x = \frac{-2}{-4} = \frac{1}{2}$$

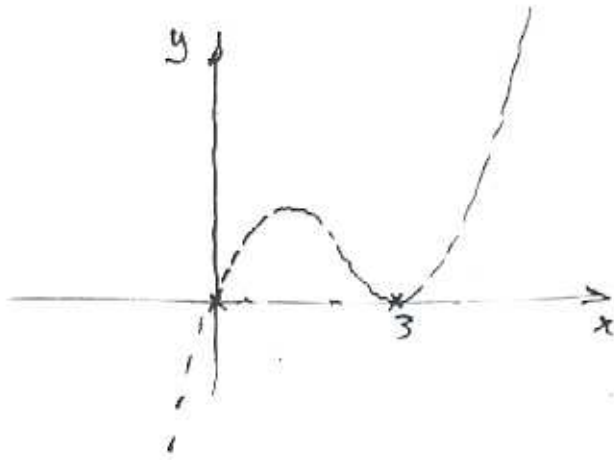
When $x = \frac{1}{2}$ $y = (\frac{1}{2})^2 - 3(\frac{1}{2}) + 4 = 2\frac{3}{4}$

graphs intersect at $\underline{\underline{(\frac{1}{2}, 2\frac{3}{4})}}$

EXTRA!!

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No. 12. (i) Sketch $y = x(x-3)^2$ roots $x=0$ $x=3$ (repeated)



ii) given $x(x-3)^2 = 2 \Rightarrow x(x^2 - 6x + 9) = 2$
 $= x^3 - 6x^2 + 9x - 2 = 0$ (as reqd.)

iii) if $x=2$ is a root then $(x-2)$ is a factor

So $x^3 - 6x^2 + 9x - 2 = (x-2)(x^2 + qx + 1)$

coeffs of x : LHS 9 = RHS $-2q + 1 \Rightarrow 2q = -8$
 $q = -4$

OR by LONG DIVISION:

$$\begin{array}{r}
 x^2 - 4x + 1 \\
 x-2 \overline{) x^3 - 6x^2 + 9x - 2} \\
 \underline{-(x^3 - 2x^2)} \\
 -4x^2 + 9x \\
 \underline{-(-4x^2 + 8x)} \\
 x - 2 \\
 \underline{-(x-2)} \\
 0
 \end{array}$$

$\Rightarrow (x-2)(x^2 - 4x + 1)$

12 (contd.)

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hence solve $(x-2)(x^2-4x+1)=0$ using formula on x^2-4x+1 ($a=1$ $b=-4$ $c=1$)

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2}$$

$$= \frac{4}{2} \pm \frac{\sqrt{12}}{2} = 2 \pm \frac{2\sqrt{3}}{2}$$

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$$

$$= 2 \pm \sqrt{3}$$

\therefore Solns (Roots) $x=2$ $x=2+\sqrt{3}$ $x=2-\sqrt{3}$

See pt (i) sketch + addition of $y=2$ and roots ...

