SECT B JUNE 2006.

11\. $A(9,8)$ $B(5,0)$, $C(3,1)$

Show AB is $\perp$r to BC.

$m_{AB} = \dfrac{8-0}{9-5} = \dfrac{8}{4} = 2$ $\qquad m_{BC} = \dfrac{1-0}{3-5} = \dfrac{1}{-2} = -\dfrac{1}{2}$

as $2 \times -\dfrac{1}{2} = -1$ [i.e. $mm' = -1$]

the AB is $\perp$r to BC.

AC is diameter of circle

mid point of AC is centre $\rightarrow \left(\dfrac{9+3}{2}, \dfrac{8+1}{2}\right) = (6, 4\tfrac{1}{2})$

length AC is $\sqrt{(9-3)^2 + (8-1)^2} = \sqrt{36+49} = \sqrt{85}$

$\therefore$ radius $= \dfrac{1}{2}\sqrt{85}$

eqn of circle $(x-6)^2 + (y-4\tfrac{1}{2})^2 = \left(\tfrac{1}{2}\sqrt{85}\right)^2$

$(x-6)^2 + (y-4\tfrac{1}{2})^2 = \dfrac{85}{4}$

(no need to simplify)

B is point $(5,0)$

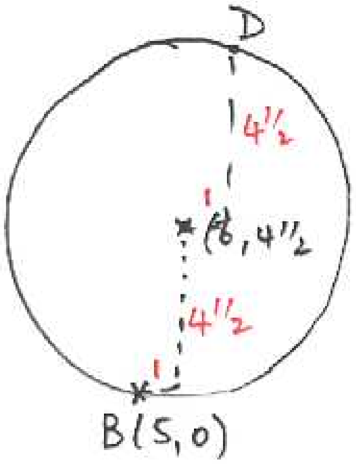
Subst. into circle eqn

$(5-6)^2 + (0-4\tfrac{1}{2})^2 = 1 + \dfrac{81}{4} = \dfrac{85}{4}$, hence B lies of circle!

BD is a diameter find coordinates of D

(see next page!)

11 (iii)



hence $D(7, 9)$

or

B to Centre $\begin{pmatrix} 1 \\ 4\frac{1}{2} \end{pmatrix}$

Centre of D also $\begin{pmatrix} 1 \\ 4\frac{1}{2} \end{pmatrix}$

$\therefore$ B to D is $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$

$(5, 0) + \begin{pmatrix} 2 \\ 9 \end{pmatrix} = (7, 9)$

.... there are other ways !!

12. (i) given $f(x) = x^3 + 9x^2 + 20x + 12$

show $x = -2$ is a Root of $f(x) = 0$

$$f(-2) = (-2)^3 + 9(-2)^2 + 20(-2) + 12$$
$$= -8 + 36 - 40 + 12$$
$$= -58 + 58 \quad = \quad 0.$$ hence $x = -2$ is a Root!

(ii) divide $f(x)$ by $x+6$.

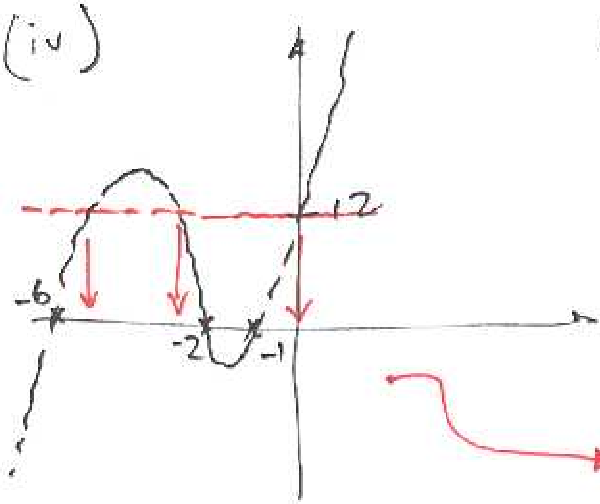
$$\begin{array}{r} x^2 + 3x + 2 \\ x+6 \overline{) x^3 + 9x^2 + 20x + 12} \\ -(x^3 + 6x^2) \\ \hline 3x^2 + 20x \\ -(3x^2 + 18x) \\ \hline 2x + 12 \\ -(2x + 12) \\ \hline \end{array}$$

remainder

ans: $x^2 + 3x + 2$

(iii) $f(x) = (x+6)(x^2 + 3x + 2)$
$$= (x+6)(x+2)(x+1)$$

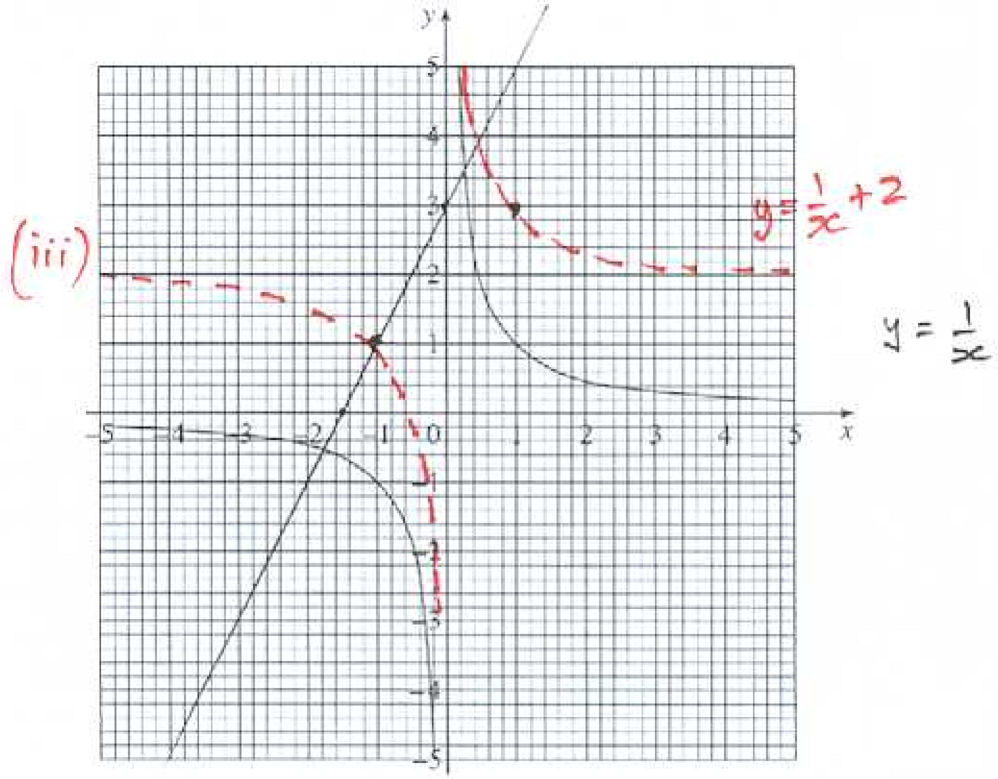
(iv)



(v) Solve $f(x) = 12$

$$x^3 + 9x^2 + 20x + 12 = 12$$
$$x^3 + 9x^2 + 20x = 0$$
$$x(x^2 + 9x + 20) = 0$$
$$x(x+5)(x+4) = 0$$
$$x = 0, -5, -4.$$

13 (i) and (iii)



(i) plot $y = 2x + 3$

(iii) ∴ Roots (from crossing pts) $x \simeq -1.8$ and $x \doteq 0.15$

(ii) $\frac{1}{x} = 2x + 3$ $(\times x) \Rightarrow 1 = 2x^2 + 3x$

$2x^2 + 3x - 1 = 0$

with $a = 2$ $b = 3$ $c = -1$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-1)}}{4}$

$= \dfrac{-3 \pm \sqrt{17}}{4}$ (leave in this form)!

(iv)

from your graph $x = -1$ and $x = 0.5$