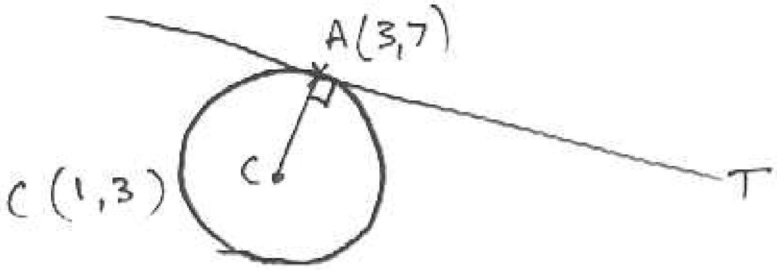
C1 June 2007

Sect B.

11.



(i) show eq^n of tangent to circle thro' A(3,7) is $x + 2y = 17$

putting (3,7) into the eqn ONLY shows that it lies on the line! (e.g. $3 + 2\times 7 = 17$)

if CA has a gradient of 2 then using $mm' = -1$ will prove radius is $\perp r$ to line thro' A AND therefore

SOLUTION !!

So....

$$m_{AC} = \frac{7-3}{3-1} = \frac{4}{2} = 2$$

the line T thro' pt. A is $x + 2y = 17$

re-arrange... $2y = 17 - x$

$$y = \frac{17}{2} - \frac{x}{2}$$

$$\therefore m_{AT} = -\tfrac{1}{2}$$

as $m_{AC} \times m_{AT} = 2 \times -\frac{1}{2} = -1$

then line AT ($x + 2y = 17$) is a TANGENT!!

11 (ii) $y = 2x - 9$ intersects tangent at T

So... $y = 2x - 9$ intersects $x + 2y = 17$ {Solve sim. eqns}

$\therefore$ by substitution...

$$x + 2(2x - 9) = 17$$

$$5x - 18 = 17 \qquad 5x = 35$$

$$x = 7$$

Subst $x = 7$ into $y = 2x - 9 \Rightarrow y = 2 \times 7 - 9 = 5$

$\therefore$ T $(7, 5)$

(iii) Substitute $y = 2x - 9$ into

$$(x-1)^2 + (y-3)^2 = 20$$

$$\Rightarrow (x-1)^2 + (2x - 9 - 3)^2 = 20$$

$$x^2 - 2x + 1 + 4x^2 - 48x + 144 = 20$$

$$5x^2 - 50x + 125 = 0$$

$$5(x^2 - 10x + 25) = 0$$

$$5(x-5)(x-5) = 0$$

$\Rightarrow$ only one soln., $x = 5$, $\therefore$ line touches circle

$\Rightarrow$ TANGENT

Subst. $x = 5$ into $y = 2x - 9$

$$y = 10 - 9 = 1$$

TANGENT touches at $(5, 1)$

12 (i) $4x^2 - 24x + 27$

$\rightarrow \quad 4\left[x^2 - 6x + {}^{27}\!/_4\right]$

$$4\left[(x-3)^2 \underbrace{-\,9}_{-9} + {}^{27}\!/_4\right] = 4\left[(x-3)^2 - \frac{9}{4}\right]$$

-9 cancels the plus 9 from expanding

$$= \underline{\underline{4(x-3)^2 - 9}}$$

$\therefore$ Minimum point is $\underline{\underline{(3, -9)}}$

(iii) Solve $4x^2 - 24x + 27 = 0$

using pt (i)

$$4(x-3)^2 - 9 = 0$$

$$4(x-3)^2 = 9$$

$$(x-3)^2 = \frac{9}{4}$$

$$x - 3 = \pm\sqrt{{}^{9}\!/_4} \quad \left(\pm\, {}^{3}\!/_2\right)$$

$$x = 3 \pm {}^{3}\!/_2$$

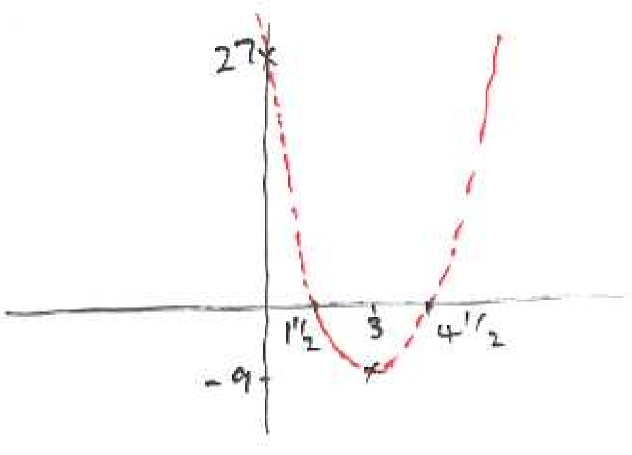
$$x = \underline{\underline{4\tfrac{1}{2} \text{ and } 1\tfrac{1}{2}}}$$

alternatively ...

$$4x^2 - 24x + 27 = (2x-3)(2x-9)$$

$$\therefore (2x-3)(2x-9) = 0 \rightarrow x = {}^{3}\!/_2 \text{ and } {}^{9}\!/_2$$

$$= \underline{\underline{1\tfrac{1}{2} + 4\tfrac{1}{2}}}$$



13. $f(x) = 2x^3 - x^2 - 11x - 12$

(i) $(x-3)(2x^2+5x+4)$ (by multiplying out...)

$2x^3 + 5x^2 + 4x - 6x^2 - 15x - 12$

$= 2x^3 - x - 11x - 12$ (as reqd.)

$f(x) = (x-3)(2x^2+5x+4)$

$f(3) = 0$ $\quad x = 3$ is a root !!

Solving $2x^2 + 5x + 4 = 0$ will find the other roots !!

(if there are any)

$a = 2 \quad b = 5 \quad c = 4$

$b^2 - 4ac = 25 - 4 \times 2 \times 4 = \underline{-7}$

$b^2 - 4ac < 0 \Rightarrow$ NO OTHER ROOTS !!

hence $x = 3$ is the ONLY ROOT !

(ii)

$f(x) = -22 \Rightarrow f(x) + 22 = 0$

Solving... $2x^3 - x^2 - 11x - 12 + 22 = 0$

Search.. $f(2) = 2(2)^3 - 2^2 - 22 - 12 + 22$

$= 16 - 4 - 22 - 12 + 22 = 0$

So $x - 2$ is a factor $x = 2$ is a Root

(iii) the last part is tricky ...

your teacher will explain !!