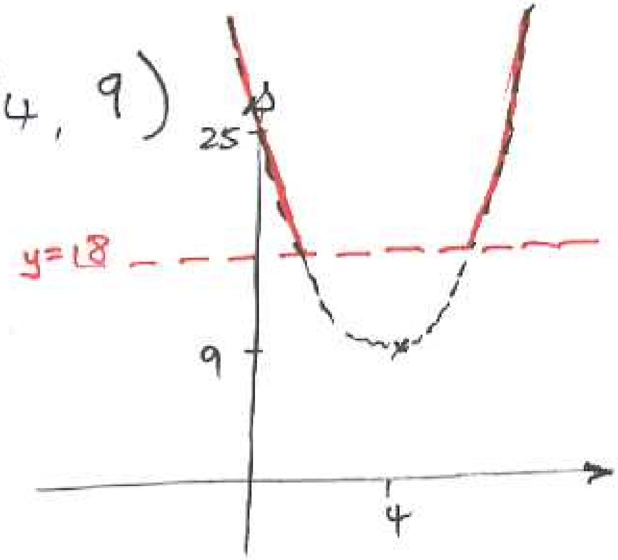
SECT. B. MAY 2005

10.(i) $x^2 - 8x + 25$ in form $(x-a)^2 + b$

$$(x-4)^2 - 16 + 25$$

$$= \underline{(x-4)^2 + 9}$$

(ii) $\therefore$ minimum point is $(4, 9)$

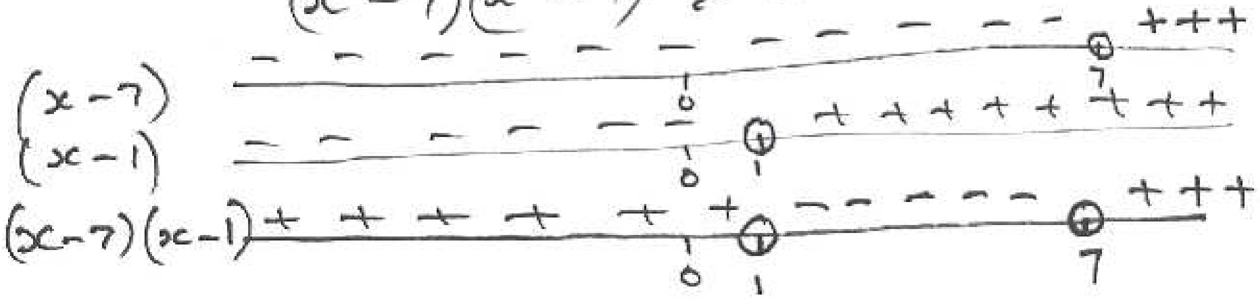


Solve $x^2 - 8x + 25 > 18$

$$x^2 - 8x + 25 - 18 > 0$$

$$x^2 - 8x + 7 > 0$$

$$(x-7)(x-1) > 0$$



$\therefore (x-7)(x-1) > 0$ when $\underline{x < 1 \text{ and } x > 7}$

$y = x^2 - 8x + 25$ is translated by $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$

$f(x) - 20$
[down 20]

eqn of new graph $y = x^2 - 8x + 25 - 20$

$$\therefore \underline{y = x^2 - 8x + 5}$$

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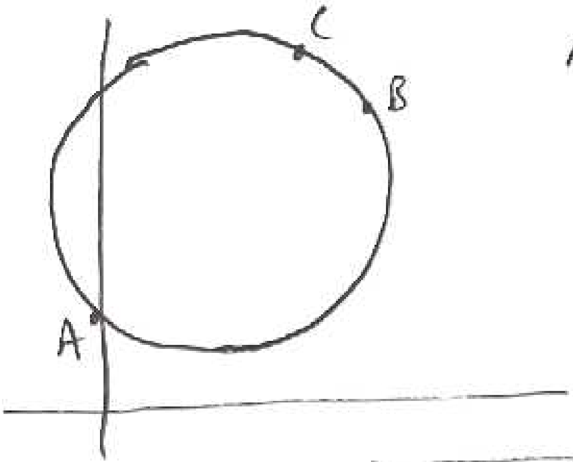
11.



A (0, 2)   B (7, 9)   C (6, 10)

using Pythagoras !!

length of AC $= \sqrt{(10-2)^2+(6-0)^2}$
$= \sqrt{8^2+6^2} = \sqrt{100} = \underline{\mathbf{10}}$

length of CB is $\sqrt{(10-9)^2+(6-7)^2} = \sqrt{2}$
" " AB is $\sqrt{(7-0)^2+(9-2)^2} = \sqrt{98}$

rt. angle at B gives $AC^2 = AB^2 + BC^2$
$\rightarrow \quad 10^2 = (\sqrt{98})^2 + (\sqrt{2})^2$
$= 98 + 2 = 100 = 10^2$

$\therefore$ right angled at B.

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if rt. angled at B then AC is a diameter
(angle in semi circle theorem)!

if AC is a diameter Centre is mid pt of AC

$\rightarrow \left(\frac{0+6}{2}, \frac{2+10}{2}\right) = (3, 6)$

radius is $\frac{1}{2}|AC| = 5$ cm.

with centre (3, 6) and radius 5

eqn of circle is $(x-3)^2 + (y-6)^2 = 5^2$

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11. (iii)

Find eqn of tangent at C (6, 10)

gradient of AC is $\frac{10-2}{6-0} = \frac{8}{6} = \frac{4}{3}$

hence using $mm' = -1$ gradient of tangent is $-\frac{3}{4}$

with $m = -3/4$ and C (6, 10)

eqn of tangent $y - 10 = -\frac{3}{4}(x - 6)$

$$y - 10 = -\frac{3x}{4} + \frac{18}{4} \qquad (\times 4)$$

$$4y - 40 = -3x + 18$$

$$\underline{\underline{3x + 4y = 58}}$$

intersects with axes.

when $x = 0$ $4y = 58$ $y = 14\frac{1}{2}$ $(58/4)$

$y = 0$ $3x = 58$ $x = 17\frac{1}{3}$ $(58/3)$

$$\underline{\underline{(17\tfrac{1}{3}, 0) \text{ and } (0, 14\tfrac{1}{2})}}$$

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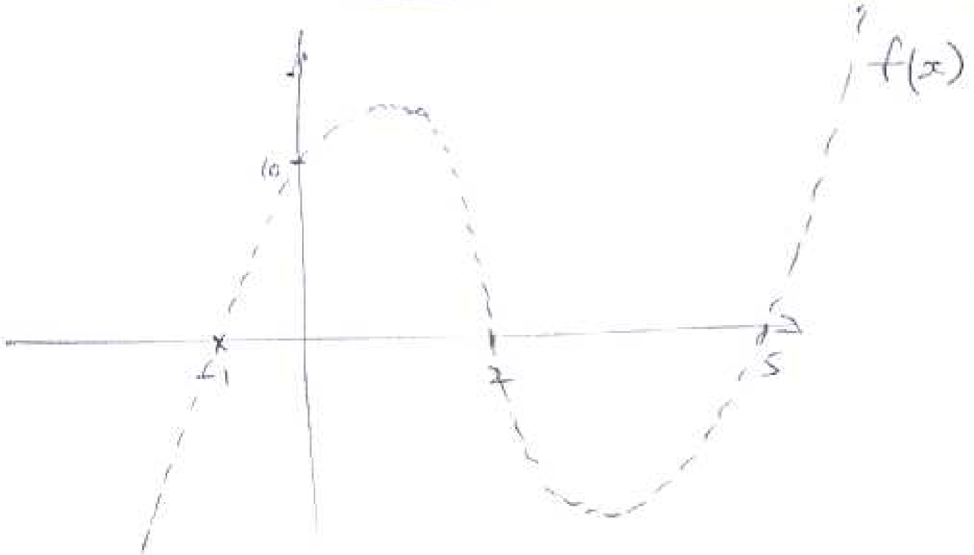
12. $f(x)$ is a cubic ; coeff of x^3 is 1

Roots of $f(x)=0$ are $-1, 2, 5$.

$$\therefore f(x) = (x+1)(x-2)(x-5)$$
$$= (x+1)(x^2 - 7x + 10)$$
$$= x^3 - 7x^2 + 10x + x^2 - 7x + 10$$
$$\therefore f(x) = x^3 - 6x^2 + 3x + 10 \quad \text{(as reqd.)}$$



$$f(x) + 10 = x^3 - 6x^2 + 3x + 20$$

if $x = 4$ $\quad f(x)+10 = 0$ if 4 is a ROOT

Subst. $\quad (4)^3 - 6(4)^2 + 3\times 4 + 20$

$$= 64 - 96 + 12 + 20 = 0 \text{ ; hence } x=4 \text{ is a Root.}$$

either divide $x^3 - 6x^2 + 3x + 20$ by $(x-4)$

OR $\quad x^3 - 6x^2 + 3x + 20 = (x-4)(x^2 + qx - 5)$

Check coeffs of x LHS $\quad 3 = $ R.H.S. $-4q - 5 \Rightarrow 4q = -8$

$q = -2$

hence quadratic is $x^2 - 2x - 5$.